BUCKLING ANALYSIS OF COMPOSITE TRUNCATED CONICAL SHELLS UNDER MECHANICAL LOADING IN THERMAL ENVIRONMENTS

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Abstract - In this paper a finite element method for investigating of thermo-mechanical buckling behavior of composite truncated conical shells is presented. In this study it is assumed that the conical shells subjected to mechanical and thermal loading simultaneously. Displacement field is based on the first shear deformation theory (FSDT). Using the principle of minimum total potential energy, stiffness matrix and geometric stiffness matrix are derived. With solving Eigen-value problem stability region identified. After validating the results of developed FE code with those available in the literature, the effect of fiber angles, thickness to radius ratio and semi vertex angle on thermo-mechanical buckling behavior of truncated conical shells is studied.

Keywords - Buckling; Thermal environments; Composite materials; Truncated conical shells; Finite element method.

I. INTRODUCTION

Thermo-mechanical buckling has an important role in the design of structures under mechanical load subjected to thermal environment. The analysis of shells of revolution such as conical shells for their applications in engineering fields, i.e. aerospace equipment and high speed rockets are immensely interesting subjects for researchers. Since composite materials are widely used because of their structural advantages, mathematical modeling of such structures under thermal and mechanical loading conditions is of prime interest and importance. Bhangale et.al [1] studied the thermal buckling and vibration behavior of truncated FGM conical shells in a high-temperature environment. Thermal buckling behavior of laminated cross-ply oval cylindrical shells based on higher-order theory was analyzed by Patel et.al [2]. Shadmehri et.al [3] presented a semi-analytical approach for obtaining the linear buckling response of conical composite shells under axial compression load. Patel et.al [4-5-6] investigated the post-buckling characteristics of the angle-ply laminated composite conical shells subjected to the torsion, external pressure, axial compression, and thermal loading considering uniform temperature change using the semi-analytical finite element approach. Ghasemi et.al [7] studied the buckling behavior of grid stiffened composite conical shells under axial loading. A study of combined thermal and mechanical buckling of stiffened cylindrical shells using the structural geometry of a preliminary supersonic transport fuselage was presented by Johnson et.al [8]. Buckling analysis of composite laminates for critical thermal and thermo-mechanical loads was reported by Jameel et.al [9]. Buckling phenomenon for static and dynamic loading of FGM plates subjected to simultaneous action of one directional compression and thermal field was presented by Michalska et.al [10]. Abolghasemi et.al [11] were investigated buckling of functionally graded plates with an elliptical cutout under combined thermal and mechanical loads using Finite Element Method. With notice to above literature review one can see that the investigation of thermo-mechanical buckling of composite truncated conical shells was not done. In this paper truncated conical shells is modeled by quadratic three nodes elements. Each node has five degree of freedoms based on FSDT. The effect of various parameters on the thermo-mechanical buckling behavior of truncated conical shells is studied and useful conclusions are presented.

II. FORMULATION

Figure 1 shows the schematic of truncated conical shells in cylindrical coordinates. In this figure L is length and R_1 , R_2 are small and large radius of conical shell respectively. β is the semi vertex angle.

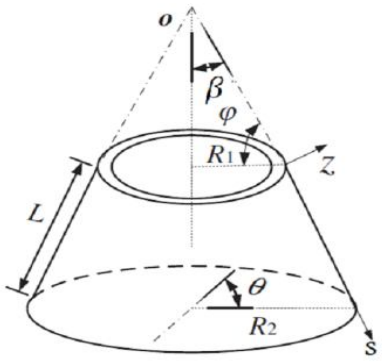


Fig. 1 Schematic view of truncated conical shells.

Displacement field base on FSDT in cylindrical coordinate (s,θ,z) as follow

$$\begin{aligned} u(s,\theta,z) &= u_0(s,\theta) + z\psi_s(s,\theta) \\ v(s,\theta,z) &= v_0(s,\theta) + z\psi_\theta(s,\theta) \\ w(s,\theta,z) &= w_0(s,\theta) \end{aligned} \quad (1)$$

u_0, v_0, w_0 are the component of displacement vector on the mid-surface. ψ_s, ψ_θ depict the rotations of tangent to mid-surface along parameter lines (s,θ). In cylindrical coordinates (s,θ,z) strain-displacement relationships based on FSDT as follow [6]

$$\begin{aligned} \varepsilon_s &= \frac{\partial u_0}{\partial s} + z\frac{\partial \psi_s}{\partial s} + \frac{1}{2}\left(\frac{\partial w_0}{\partial s}\right)^2 \\ \varepsilon_\theta &= \frac{1}{r}\left(u_0\sin\beta + \frac{\partial v_0}{\partial \theta} + w_0\cos\beta\right) + \\ & z\frac{1}{r}\left(\psi_s\sin\beta + \frac{\partial \psi_\theta}{\partial \theta}\right) + \frac{1}{2}\left(\frac{1}{r}\left(\left(\frac{\partial w_0}{\partial \theta} - v_0\cos\beta\right) + z\left(-\psi_\theta\cos\beta\right)\right)\right)^2 \\ \gamma_{s\theta} &= \frac{1}{r}\left(\frac{\partial u_0}{\partial \theta}\right) + \left(\frac{\partial}{\partial s} - \frac{\sin\beta}{r}\right)v_0 + \\ & \left(\frac{1}{r}\left[\left(\frac{\partial w_0}{\partial \theta} - v_0\cos\beta\right) + z\left(-\psi_\theta\cos\beta\right)\right]\right)\left(\frac{\partial w_0}{\partial s}\right) \\ \gamma_{sz} &= \frac{\partial w_0}{\partial s} + \psi_s \\ \gamma_{z\theta} &= \frac{1}{r}\left(\frac{\partial w_0}{\partial \theta} - v_0\cos\beta\right) + \psi_\theta \end{aligned} \quad (2)$$

where r as follow

$$r = R_1 + s(sin\beta) \quad (3)$$

With notice to above equations, relationship between forces and moments with strains are as follows

$$\{N_s\ N_\theta\ N_{s\theta}\ M_s\ M_\theta\ M_{s\theta}\ Q_s\ Q_\theta\}^T = [D]\{\varepsilon_t\} \quad (4)$$

$[D]$ is the constitutive matrix and $\{\varepsilon_t\}$ is the total linear strain vector that consists of mechanical and thermal components.

In order to axisymmetric properties of conical shells, temperature distribution is depended to circumferential direction according to Fourier series as follow

$$\Delta T = \sum_{n=0}^{\infty} \Delta T_n \cos(n\theta) \quad (5)$$

Potential energy of elastic structure can be addressed as follow

$$\Pi = \frac{1}{2}\int_v \left\{\{\varepsilon_t\}^T\{\sigma\} + \{\varepsilon^{NL}\}^T\{\sigma^*\}\right\}dv - \{F_e^m\}\{d_e\} \quad (6)$$

$\{\sigma\}, \{\varepsilon^{NL}\}, \{\sigma^*\}, \{F_e^m\}$ are the stresses, nonlinear strains, initial stresses and mechanical loading vectors, respectively.

Linear and nonlinear strains versus displacements vector expressed as

$$\left(\{\varepsilon\}, \{\varepsilon^{NL}\}\right) = \left([B], [B_{NL}]\right)\{d\} \quad (7)$$

where $[B], [B_{NL}]$ are the strain-displacement matrices.

Using the Hooke's law and integration on the z direction, potential energy can be rewritten as

$$\begin{aligned} \Pi = &\frac{1}{2}\{d_e\}^T[K_e]\{d_e\} + \frac{1}{2}\{d_e\}^T[K_{Ge}]\{d_e\} \\ &-\{d_e\}^T\left\{\{F_e^{th}\} + \{F_e^m\}\right\} \end{aligned} \quad (8)$$

Element stiffness matrix can be derived from above equation

$$[K_e] = \int_A [B]^T[D][B]rdsd\theta \quad (9)$$

Also, element thermal loading vector is

$$\{F_e^{th}\} = \int_A [B]^T[D]\{\varepsilon_0\}rdsd\theta \quad (10)$$

where $\{\varepsilon_0\}^T = \Delta T\{\alpha_s, \alpha_\theta, \alpha_{s\theta}, 0, 0, 0, 0, 0\}$

$[K_{Ge}]$ is the element geometric stiffness matrix and can be expressed as

$$[K_{Ge}] = \int_A [B_{NL}]^T[N][B_{NL}]rdsd\theta \quad (11)$$

where

$$[N] = \begin{bmatrix} N_s & N_{s\theta} \\ N_{s\theta} & N_\theta \end{bmatrix} \quad (12)$$

According to the principle of minimum total potential energy standard equation of finite element method is derived as follows

$$[K_e]\{d_e\} + [K_{Ge}]\{d_e\} = \{F_e\} \quad (13)$$

$\{F_e\}$ is the summation of thermal and mechanical loading.

With assembling the matrices and vectors for linear static analysis, we have

$$[K]\{d\} = \{F\} \quad (14)$$

and the buckling analysis equations will be

$$\begin{aligned} &\left|[K] + \Delta T\left[K_G^*\right]_{th}\right|\{d\} = 0 && \text{Thermal buckling} \\ &\left|[K] + \lambda\left[K_G^*\right]_{mech}\right|\{d\} = 0 && \text{Mechanical buckling} \end{aligned} \quad (15)$$

$\left[K_G^*\right]_{th}$ is the assemble geometric stiffness matrix for unit temperature rise. $\left[K_G^*\right]_{mech}$ is the assemble geometric stiffness matrix for unit mechanical loading rise and λ is the mechanical loading coefficient.

In thermo-mechanical buckling analysis, we assume that the shell is exposed to initial loading. This initial loading can be mechanical or thermal loading. For the shell with specified initial mechanical loading, one can find the temperature of thermo-mechanical buckling. For this purpose, Eq.(14) is used for finding the pre-buckling displacement. In thermo-mechanical loading, displacement vector is included two sections. First section associated to thermal loading for unit temperature rise and can be calculated as

$$\{d_{th}\} = [K]^{-1}\{F_{th}\} \quad (16)$$

second section of displacement vector due to the mechanical loading in edge of the shell and can be achieved as

$$\{d_{mech}\} = [K]^{-1} \lambda \{F^0_{mech}\} \quad (17)$$

$\{F^0_{mech}\}$ is the unit initial mechanical load. Since the governing prebuckling equations is the linear with applying the superposition principle, we have

$$\{d\} = \Delta T \{d_{th}\} + \{d_{mech}\} \quad (18)$$

After calculating the prebuckling displacement vector, we can be find the critical buckling temperature as follow

$$([K] + [K_G])\{d\} = 0 \quad (19)$$

The geometric stiffness matrix is the function of prebuckling displacement and consists of two sections as follow

$$\left([K] + [K_G]_{mech} + \Delta T [K^*_G]_{th}\right)\{d\} = 0 \quad (20)$$

For solving the Eq.(20) we have the Eigen-value problem as follow

$$\left|([K] + [K_G]_{mech}) + \Delta T [K^*_G]_{th}\right|\{d\} = 0 \quad (21)$$

For specified mechanical load, one can see that the critical buckling temperature is the response of the above Eigen-value problem.

RESULT AND DISCUSSION

Tables 1 and 2 show the material and geometrical properties, respectively. Conical shell made of composite with four layers as [90˚/45˚/-45˚/90˚]. Temperature environment is assumed 20 ˚C.

The vertex angle is considered 45˚. In presentation of boundary conditions, left and right character is related to small and large end of conical shells, respectively.

Table 1. Material properties

181	E_1 (MPa)
10.34	E_2, E_3 (MPa)
7.2	G (MPa)
0.28	ν
11.3×10^{-6}	$\alpha_1(1/^{\circ}C)$
36.9×10^{-6}	$\alpha_2(1/^{\circ}C)$

S1 and S2 boundary conditions are as follow

$$S1: u_0 = 0 \quad v_0 = 0 \quad \psi_\theta = 0 \quad w_0 = 0$$
$$S2: v_0 = 0 \quad \psi_\theta = 0 \quad w_0 = 0 \quad (28)$$

$T_c = \alpha_2 \times 10^3 \times \Delta T_{cr}$ is presented dimensionless thermal critical buckling temperature and $N_c = \dfrac{N_{x,cr}\left(1-\nu_1^2\right)}{E_1 h}$ is presented dimensionless mechanical critical buckling load.

Table 2. Geometry properties

B.C	(L/R_1)	h (m)	R_1 (m)
S1-S2	1.5	0.003	0.3

Investigation of convergence of results, shows that stability diagram for 50 elements are appropriate.

In Table 3 presented results for thermal buckling is verified with reference [5].The effect of length of shells on the buckling behavior of composite truncated conical shells is showed in Figure 2. Increase in length due to smaller stability region.

Table 3. Comprision of non-dimensional thermal buckling temperature for various vertex angle

β	Patel [5]	Present	Error%
0˚	0.1034	0.1018	0.4
15˚	0.08904	0.08917	0.14
30˚	0.07496	0.07458	0.6
45˚	0.05951	0.05977	0.5
60˚	0.044	0.0438	0.3

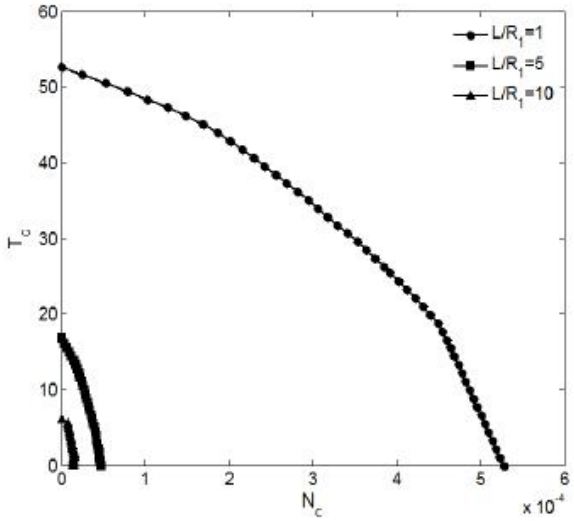
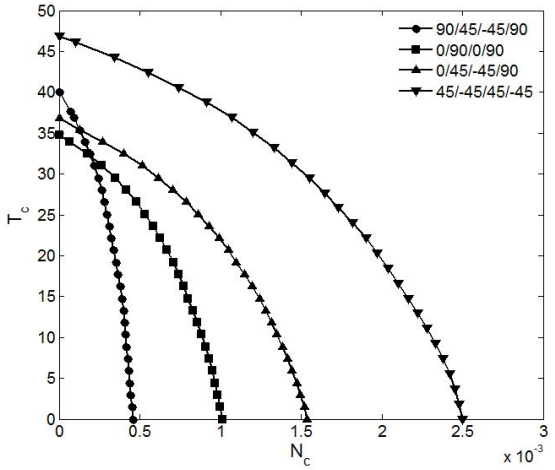


Fig. 2 Investigation of the effect of length on the thermo-mechanical buckling.

One can see that in Figure 3 the stability region of thermo-mechanical buckling is affected from stacking sequence, extremely.



According to Figure 4 for smaller vertex angle, stability region of thermo-mechanical buckling is bigger. In other word, with change in geometric of

structure from cylindrical shell to annular perforated plate amount of resistance to buckling decreases.

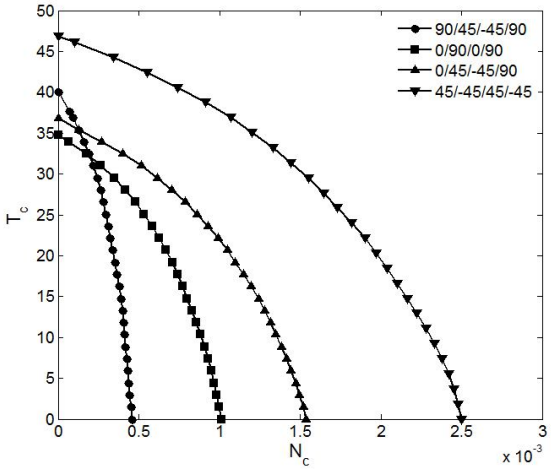


Fig. 4 Investigation of the effect of vertex angle on the thermo-mechanical stabiity region.

CONCLUSION

In this paper thermo-mechanical buckling of truncated conical shells with semi-analytical finite element is investigated. The effects of vertex angle, stacking sequence and length to radius aspect ratio of shells on the stability region are studied. The following beneficial results as obtained at the end.

1. The longer shells has smaller stability region.
2. Cylindrical shells, stability region of thermo-mechanical buckling is bigger than those annular perforated plates.
3. Stacking sequence has very important role in the scope of stability region.

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