ACCURACY OF ROTATION-ANGLE OF SOFTENED TRUSS MODEL FOR REINFORCED CONCRETE BEAMS UNDER PURE TORSION

[1]SEUNGEUN KIM, [2]JUNG-YOON LEE, [3]CHANG-HYUK KIM

Department of Civil, Architectural and Environmental System Engineering, Sung Kyun Kwan University
E-mail: [1]se21008@naver.com, [2]jungyoon@skku.edu, [3]changhyuk@utexas.edu

Abstract - The Rotating-Angle of softened Truss Model (RA-STM) by Thomas T. C. Hsu and Y. L Mo, developed for predicting the post-cracking torsional behavior of reinforced concrete beams up to the ultimate torsional strength, takes account two of Equilibrium equations and the softened constitutive equations for concrete struts in bending and compression. In this paper, the ultimate torsional strength and failure modes of 10 beams were observed by using RA-STM. Specimens have different yield strength of torsional steel and the ratio of amount of reinforcement to the maximum amount of reinforcement of ACI 318-R14. Especially a correct behavior prediction of the specimens using high-strength reinforcement or having the maximum amount of reinforcement of ACI 318-R14($\rho_t f_{yt} / \rho_t f_{yt,max}$) is important as design codes including the ACI 318-14 design code and Eurocode 2 limit the yield strength and the maximum amount of torsional reinforcement in order to avoid the brittle failure and induce torsional tension failure.

Index Terms - Rotating-Angle of Softened Truss Model, ultimate torsional strength, high-strength reinforcement, failure mode

I. INTRODUCTION

Recently, the use of high-strength structural materials has increased due to increasing demand of high-rise buildings and large-scale structures. In particular, using high strength concrete can reduce the size of the members and make more space of structures. When using high strength concrete, the amount of reinforcement needs to be increased in order to improve the resistance. There are two ways of increasing the amount of reinforcement (ρf_y). One is to increase the yield strength of reinforcement (f_y) and the other is to increase the ratio of reinforcement (ρ). However, the design codes in many countries limits the yield strength of torsional reinforcement and the amount of reinforcement for structural safety.
For example, ACI 318-14 [1] and Eurocode 2 [2] limit the yield strength of torsional reinforcement to 420MPa, and to 600MPa respectively, to lead the torsional reinforcement to yield and control the distance of diagonal cracks. The maximum amount of reinforcement is restricted for the reason to prevent the brittle compression failure. As the purpose of each code's restriction is to control the failure modes, it is important to validate if the restriction would work properly.
In the present paper, the accuracy of the ultimate torsional strength and failure modes were studied by comparing the test result to analytical value of the RA-STM and to the ACI predictions. Ten beams were observed. Specimens have different yield strength of torsional steel and the ratio of maximum amount of reinforcement of ACI 318-R14.

II. TEST PROGRAM

A. Test Plan

Test specifics of 10 specimens are on the Table I. The specimens are divided into four groups. The group 1 and the group 2 are the beams that the amount of torsional reinforcement to the maximum amount of reinforcement of ACI 318-R14 are 80 percent and 100 percent each. The group 3 and the group 4 are beams with the same space of transverse steel, which are 90mm and 72.5 mm respectively.
Fig. 1 shows the section and the configuration of specimens. Specimens are consisted of a 2000mm-long beam with a 500mm-long stub. The cross section of the beam is 400x600mm. The strength of concrete is 35.4MPa. The main variable of the test is strength of steel and the ratio of the amount of reinforcement to the maximum amount of reinforcement of ACI 318-R14. Three different kinds of yield strength of steel are used. (about 350MPa, 480 MPa and 660MPa) The group 1 and 2 are consisted with the three beams made of yield strength of reinforcement of 350MPa, 480MPa and 660MPa each, while the third and fourth group are consisted with two beams using a yield strength of reinforcement of 350MPa and 660MPa. The ratio of longitudinal reinforcement to transverse reinforcement were set up to be close to 1.0 so that the angle of diagonal concrete strut becomes 45 degree. (3) The stub was fixed to the reaction bottom using hydraulic jacking device. The torque arm was located in the distance of 375mm from the end of the test zone. The length of torque arm is 700mm.

B. Test Measurement Plan

To know the failure modes of the specimens it is essential to indicate if the steel yields before the ultimate torsional strength. To measure strain of steel, strain gauges were attached to the longitudinal and transverse reinforcements.
In longitudinal direction, four gauges were attached in top, middle and bottom side of reinforcements at

250mm, 520mm, and 780mm and 1030mm points from the start of the test zone. [3] In transverse direction, strain gauges were attached to the top and left side of every two transverse reinforcements. The location of gauges are marked with white squares in Fig. 2. Two LVDTs were set to the left and the right end of the test zone to check the angle of twist.

Specimen.			Longitudinal reinforcement			Transverse reinforcement				Amount of Reinforcement	
		f_{ck}	d_l	f_{yl}	ρ_l	d_t	s	f_{yt}	ρ_t	$\rho_t f_{yt}$	$\rho_t f_{yt} / \rho_t f_{yt,max}$
		MPa	ea-mm	MPa	-	mm	mm	MPa	-	MPa	
Group 1	T1-350-65	35.4	4-D16+8-D13	313.34	0.00753	D10	65	334.18	0.00750	2.50	0.76
	T1-480-90	35.4	6-D13+8-D10	474.64	0.00554	D10	90	485.74	0.00541	2.63	0.82
	T1-660-122.5	35.4	6-D13+6-D10	568.79	0.00436	D10	122.5	594.96	0.00398	2.37	0.76
Group 2	T2-350-52.5	35.3	10-D16+2-D13	320.20	0.00933	D10	52.5	334.18	0.0928	3.10	0.95
	T2-480-72.5	35.3	12-D13+2-D10	467.98	0.00693	D10	72.5	485.74	0.00672	3.26	1.02
	T2-660-100	35.3	8-D13+4-D10	566.87	0.00541	D10	100	594.96	0.00487	2.90	0.94
Group 3	T3-350-90	35.4	6-D13+8-D10	318.43	0.00554	D10	90	334.18	0.00541	1.81	0.56
	T3-660-100	35.4	8-D13+6-D10	569.65	0.00601	D10	90	594.96	0.00541	3.22	1.04
Group 4	T4-350-72.5	35.3	12-D13+2-D10	308.97	0.00693	D10	72.5	334.18	0.00672	2.25	0.68
	T4-660-72.5	35.3	12-D13+4-D10	564.65	0.00752	D10	72.5	594.96	0.00672	4.00	1.29

Table I. Specification of specimens and material properties in reference [3]

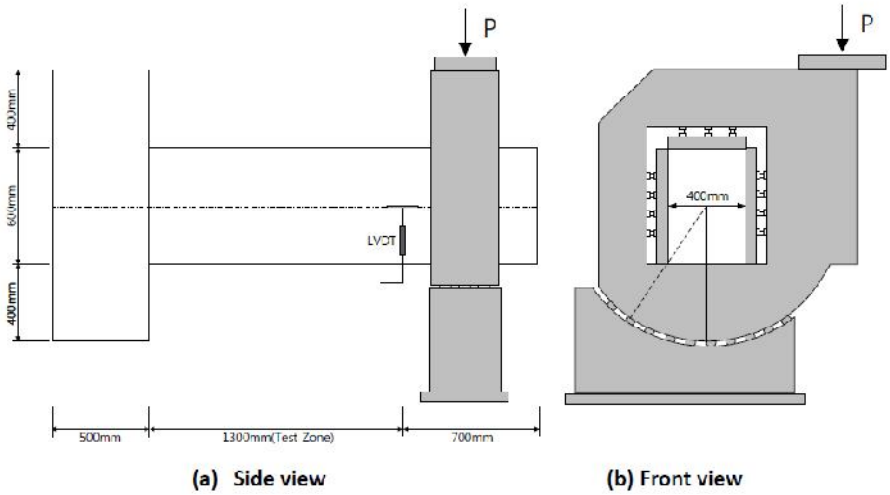


Fig. 1 Overall dimension of beams and location of LVDT in reference [3]

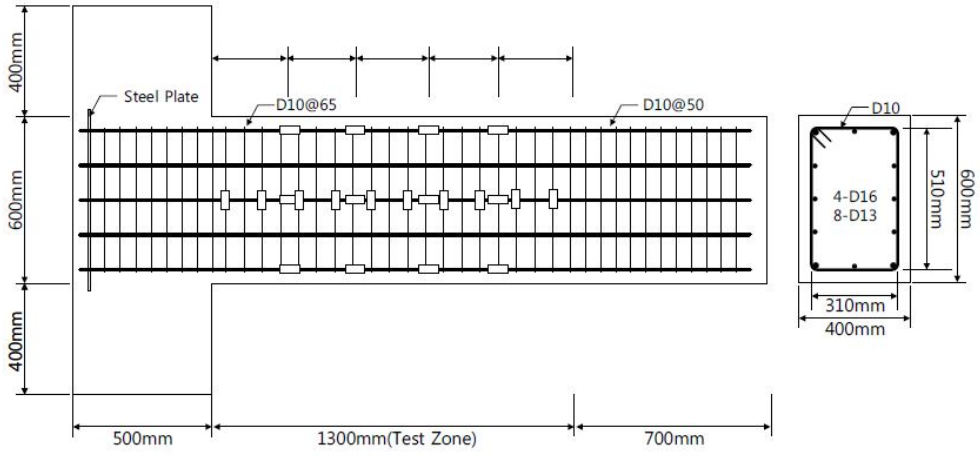


Fig. 2 Configuration of specimen and location of strain gauges of T1-350-65 in reference [3]

III. TEST RESULTS

A. Torque-Twist Curves

The relationships of Torque and twist of test are presented with solid line in Fig.5. For the comparison between the design code and the test results, the equation of ACI 318-14 is used. The equation for calculating the torsional strength in ACI 319-14 is (1), which is based on the thin-wall tube theory and the space truss model. Equation (2) determines the angle of concrete strut (α).

$$T_n = \min\left\{\frac{2A_0A_tf_{yt}}{s}\cot\alpha, \frac{2A_0A_lf_{yt}}{p_h}\cot\alpha\right\} \quad (1)$$

$$\cot^2\alpha = \frac{\rho_l f_{yl}}{\rho_t f_{yt}} \quad (2)$$

Where: A_0 is gross area enclosed by shear flow path. A_t and A_l are cross-sectional area of transverse and longitudinal reinforcement, respectively. f_{yt} and f_{yl} are the yielding strength of each transverse and longitudinal reinforcement. s stands for the space of stirrups. α is the angle of the concrete strut. ρ_l and ρ_t are mild steel ratio in the l- and t-directions respectively.

In the case of specimens using normal strength steel, the reinforcements yielded before the ultimate strength and ACI 318-14 underestimated the ultimate strength, which means the value predicted by ACI code is on safety side. The same tendency is shown on the specimens having the lower amount of reinforcement than the maximum amount of reinforcement in ACI 318-14.

The reinforcement of specimen with high-strength steel tends not to yield before the ultimate torsional strength. For this reason, the specimen T1-660-122.5 of Goup1 and T2-660-100 of Group 2 using high-strength steel shows much lower ultimate strength than T1-350-65 and T2-350-52.5, although they have the same amount of reinforcement.

The specimens that does not satisfy the maximum amount of reinforcement restriction in ACI 318-14, did not fail in tension. The value by (1), as presuming the strength of reinforced concrete beams after the yielding of reinforcement, overestimated the ultimate strength than test results. All the values of ACI calculation and the test results are presented in Table II.

	Tn,ACI	Tn,Test	Tn,RA STM	Tn,ACI/ Tn,test	Tn,RA STM/ Tn,Test	Failure Mode		
						ACI	Test	RA-STM
T1-350-65	97.52	123.45	126.23	0.79	1.02	UR	UR	UR
T1-480-90	105.71	124.11	119.96	0.85	0.97	UR	UR	UR
T1-660-122.5	97.61	88.87	124.95	1.10	1.41	UR	OR	OR
T2-350-52.5	122.28	126.11	152.16	0.97	1.21	UR	OR	UR
T2-480-72.5	130.67	110.14	161.20	1.19	1.46	OR	OR	OR
T2 660 100	120.22	109.01	149.92	1.10	1.38	UR	OR	OR
T3-350-90	71.77	101.1	96.12	0.71	0.95	UR	UR	UR
T3-660-100	133.88	117.15	163.13	1.14	1.39	OR	OR	OR
T4 350 72.5	87.85	105.61	114.66	0.83	1.09	UR	UR	UR
T4-660-72.5	166.10	119.83	175.76	1.39	1.47	OR	OR	OR
			AVE.	1.03	1.23			
			C.V.	0.21	0.20			

Table II. Ultimate strength and failure mode of test result, ACI and RA-STM

B. Failure modes

The failure modes can be determined by the state of longitudinal and transverse reinforcements. If both reinforcements have yielded prior to the ultimate strength, the failure mode of the specimen is under-reinforced failure (UR). In opposite case, the failure mode is over-reinforced failure (OR). The failure modes of ACI 318-14 is determined as OR when the amount of the reinforcement exceed the maximum amount of reinforcement, otherwise the failure mode is determined as UR. The equation (4) is for the maximum amount of ACI 318-14. It can be induced from (3) of cross-sectional limits, the empirical formula.

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + \frac{2\sqrt{f_{ck}}}{3}\right) \quad (3)$$

$$\rho_t f_{yt,max} \le \frac{5}{6}\sqrt{f_{ck}}\left(\frac{A_{oh}}{A_g}\right) \quad (4)$$

where: V_u is factored shear force at section, and b_w is web width. d is effective depth. T_u is factored torsional moment at section. A_{oh} is area enclosed by centerline of the outermost closed transverse torsional reinforcement. V_c is nominal shear strength by concrete. A_g is gross area of concrete section.

Table II shows the failure mode of all specimens. The test specimens with normal strength steel and of the lower ratio of the amount of reinforcement than 1.0 (T1-350-65, T1-480-90, T3-350-90, T4-350-72.5) failed in tension. The failure mode of specimens using high-strength steel or of the ratio of the reinforcement amount over 0.9 failed in strut-compression. This result shows the design code regulation against using high-strength steel on torsional members is reasonable. However, as the specimens with the steel amount under the maximum limitation also failed in compression, the maximum limitation value in ACI 318-14 needs to be adjusted.

IV. THE ROTATION-ANGLE OF SOFTENED TRUSS MODEL

A. Governing Equations for Torsion

A Rotation-Angle of Softened Truss Model developed by Hsu and Mo is based on thin tube theory and space truss model. It uses two compatibility equations and the softened stress-strain curves for concrete. The softening effect on concrete makes the model closer to a realistic behavior by

applying the weakened strength of concrete strut due to cracks by tension. The equations and the flow chart for the RA-STM follow below. [4]

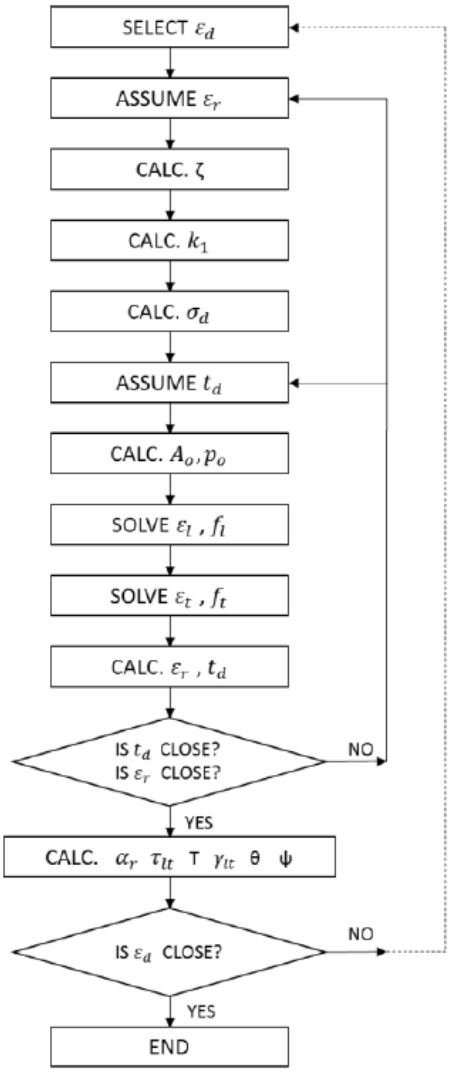


Fig. 3 Flow chart of RA-STM

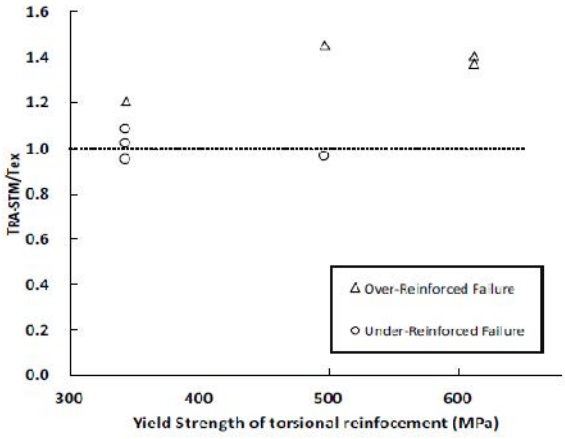


Fig. 4 Ratio of ultimate strength and yielding strength of torsional reinforcement

Equilibrium equations

$$\sigma_l - \rho_l f_l - \rho_{lp} f_{lp} = \sigma_d \sin^2 \alpha_r \quad (5)$$

$$\sigma_t - \rho_t f_t - \rho_{tp} f_{tp} = \sigma_d \cos^2 \alpha_r \quad (6)$$

$$\tau_{lt} = (-\sigma_d) \sin \alpha_r \cos \alpha_r \quad (7)$$

$$T = \tau_{lt}(2A_o t_d) \quad (8)$$

where: f_l and f_t are stresses in mild steel in the l- and t-directions, respectively. t_d is a thickness of shear flow zone and τ_{lt} is the shear stress. σ_l and σ_t are the stress in longitudinal and transverse directions. σ_d is the compressive stress in concrete.

Compatibility Equations

$$\varepsilon_l = \varepsilon_r \cos^2 \alpha_r + \varepsilon_d \sin^2 \alpha_r \quad (9)$$

$$\varepsilon_t = \varepsilon_r \sin^2 \alpha_r + \varepsilon_d \cos^2 \alpha_r \quad (10)$$

$$\frac{\gamma_{lt}}{2} = (\varepsilon_r - \varepsilon_d) \sin \alpha_r \cos \alpha_r \quad (11)$$

$$\theta = \frac{p_o}{2A_o} \gamma_{lt} \quad (12)$$

$$\psi = \theta \sin 2\alpha_r \quad (13)$$

$$t_d = \frac{\varepsilon_{ds}}{\psi} \quad (14)$$

$$\varepsilon_d = \frac{\varepsilon_{ds}}{2} \quad (15)$$

where : α_r is angle of the concrete strut, and The angle of twist θ is produced by a shear strain γ_{lt}. Ψ is a bending curvature of concrete struts. ε_{ds} is the maximum strain at the surface.

Constitutive las of concrete in compression

$$\sigma_d = k_1 \zeta f_c' \quad (16)$$

$$k_1 = \frac{\varepsilon_{ds}}{\xi \varepsilon_o}\left(1 - \frac{1}{3}\frac{\varepsilon_{ds}}{\zeta \varepsilon_o}\right) \quad \frac{\varepsilon_{ds}}{\varepsilon_o} \le 1 \quad (17)$$

$$k_1 = \left[1 - \frac{\zeta^2}{(2-\zeta)^2}\right]\left(1 - \frac{1}{3}\frac{\zeta \varepsilon_o}{\varepsilon_{ds}}\right) + \frac{\zeta^2}{(2-\zeta)^2}\frac{\varepsilon_{ds}}{\xi \varepsilon_o}\left(1 - \frac{1}{3}\frac{\varepsilon_{ds}}{\zeta \varepsilon_o}\right) \quad \frac{\varepsilon_{ds}}{\varepsilon_o} > 1 \quad (18)$$

where: k1 is the ratio of the average stress to the peak stress. ζ is the softening coefficient.

Constitutive law of mild steel

$$f_l = E_s \varepsilon_l \quad \varepsilon_l < \varepsilon_{ly} \quad (19)$$

$$f_l = f_{ly} \quad \varepsilon_l \ge \varepsilon_{ly} \quad (20)$$

$$f_t = E_s \varepsilon_t \quad \varepsilon_t < \varepsilon_{ty} \quad (21)$$

$$f_t = f_{ty} \quad \varepsilon_t \ge \varepsilon_{ty} \quad (22)$$

B. Comparison of the RA-STM with tests

The average of ultimate strength ratio ($T_{n,RA-STM}/T_{n,Test}$) of total specimens is 1.23, which means lower accuracy rate compared to ACI 318-14 ($T_{n,ACI}/T_{n,Test}$). The reason for this is that the specimens with high-strength steel and the over-reinforced specimens lower the accuracy of ultimate strength. All specimens with high-strength steel were failed in OR and showed low accuracy of ultimate strength ratio, which is already acknowledged in the paper by Lee [5]. The over-reinforced specimen also showed OR failure and low accuracy of ultimate strength. In short, all specimen of OR failure showed low accuracy on the ultimate strength, even if the RA-STM predicts the correct failure mode. The average of ultimate strength ratio ($T_{n,RA-STM}/T_{n,Test}$) of UR specimen is 1.01, which is more accurate rate compared to the

average rate of OR specimen, 1.38. Fig. 4 is the graph of the ratio of ultimate torsional strength and yield strength of torsional reinforcement. The circle marks and the square marks represents the specimen of over-reinforced failure (OR) and the under-reinforced failure (UR), respectively.

RA-STM predicts the failure mode by checking the yielding of reinforcement. The prediction of failure modes by RA-STM presents more accurate results than ACI 318-14. The RA-STM failed in failure mode prediction on one specimen, while ACI failed in three specimens. There are two reasons for this. First, the equation (4) does not reflect the limitation of maximum yielding strength of reinforcement in ACI 318-14, which is 420MPa. Second reason is that the empirical formula of ACI 318-14 underestimates the maximum amount of reinforcement, and predicts for all specimen of OR failing in UR.

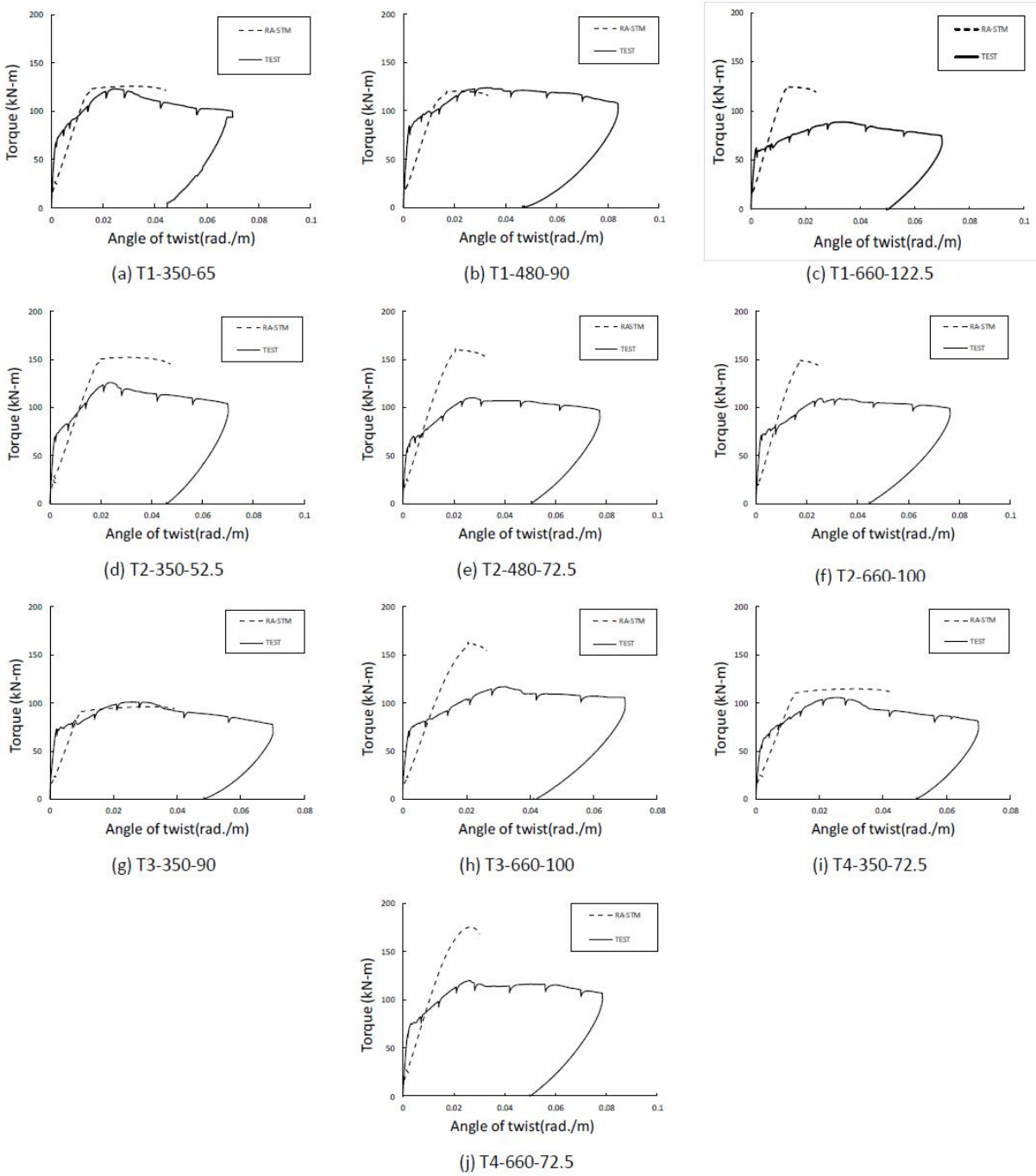


Fig. 5 The torque-twist curves of test results and RA-STM

Fig. 5 shows the torque-twist curves of test results in solid line and of RA-STM in dotted line. RA-STM cannot predict the pre-cracked branch and the angle of twist accurately because the model is based on the tube theory assuming that the specimen already cracked and ignores the tension stress of concrete. [6]

CONCLUSION

1. RA-STM shows the accurate precision to the ultimate strength of the specimens with normal stress reinforcement under the amount of maximum reinforcement. By applying the effect of softening concrete to the model, it increases the thickness of effective shear flow and decrease and lever arm, which allows the model to predict the relatively accurate strength.
2. The specimen with using more of the maximum amount of reinforcement predicted by RA-STM is not accurate as well as the ultimate strength of the specimen with high-strength steel regardless of RA-STM's correct precision on failure mode.
3. RA-STM, the theoretical equation, predicted the failure mode better than ACI 318-14, the empirical equation.
4. The prediction on pre-cracked branch of torsional behavior by RA-STM was unreliable because the model assumes the specimen cracked from the first.

ACKNOWLEDGMENT

This work was supported by the Industrial Strategic technology development program (10063488, Development of Earthquake Resisting Reinforced Concrete using grade 700MPa Reinforcing Bars for enhancement of seismic safety) funded By the Ministry of Trade, industry & Energy (MI, Korea) and by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (2016R1A6A3A11931420)

REFERENCES

[1] ACI Committee 318, "Building Cond Requirements for Structural Concrete (ACI 318-14) and," American Concrete Institute, Farmington Hills, MI: 2014, 520pp

[2] (BS EN 1992-1-1),"Eurodode 2: Design of Concrete Structures, Part 1-1: General Rules and Rules for Buildings," Comite European De Normalisation (CEN), Lausanne, Switserland, 2004, 227pp

[3] S.-H. Lee, "Torsional Resistance Mechanism of Reinforced Concrete Beams,"M.S. thesis, Dept. of Civil, Architectural and Environmental System Engineering, Sungkyunkwan Univ., Suwon, Republic of Korea, 2016.

[4] Thomas T.C. Hsu and T.L. MO, "Unified theory of concrete structures," Boca Raton, FL: CRC Press, 1993, 313pp

[5] S.-H. Lee, "Torsional Behavior of Reinforced Concrete Beams Predicted by Compatibility-Aided Truss Model," International Journal of Structural and Civil Engineering Research Vol. 5, No. 4, November 2016, 272pp

[6] Chyuan-Hwan Jeng and Thomas T.C. Hsu, "A softened membrane model for torsion in reinforced concrete members," Engineering structures 31 (2009), 1952pp

★★★