STABILITY OF HOPFIELD NEURAL NETWORKS

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Abstract - Back propagation (BP) neural network is used to approximate the dynamic character of nonlinear discrete-time system. Considering the unmodeling dynamics of the system, the weights of neural network are updated by using a dead-zone algorithm and a robust adaptive controller based on the BP neural network is proposed. For the situation that jumping change parameters exist, multiple neural networks with multiple weights are built to cover the uncertainty of parameters, and multiple controllers based on these models are set up. At every sample time, a performance index function based on the identification error will be used to choose the optimal model and the corresponding controller. Different kinds of combinations of fixed model and adaptive model will be used for robust multiple models adaptive control (MMAC). The proof of stability and convergence of MMAC are given, and the significant efficacy of the proposed methods is tested by simulation. This paper deals with the problem of delay-dependent stability criterion of delay-difference system with multiple delays of Hopfield neural networks. Based on quadratic Lyapunov functional approach and free-weighting matrix approach, some linear matrix inequality criteria are found to guarantee delay-dependent asymptotical stability of these systems. And one example illustrates the exactness of the proposed criteria.

Keywords - Hopfield neural networks; Time-varying Delay; Stability; Quadratic Lyapunov functional approach.

I. INTRODUCTION

Due to the strong ability of approximation, neural network has been widely used in the identification of nonlinear system. It is also a very useful tool for prediction, pattern recognition, and control [1]. Then network structure comprises the interconnected group of nodes and the weight. There are many kinds of neural networks such as back propagation (BP), radial basis function (RBF), cerebella model articulation controller (CMAC). As the most effective learning algorithm for feed forward networks [2], BP neural network has been the research focus for many years [3–6]. Adaptive control of nonlinear systems using neural network has been an active research area for over two decades [7–9].The controller will be set up by adjusting the weights of the neural network [10, 11]. But adaptive control using neural network still has the same shortcomings as conventional adaptive control; it is extensively studied in time-invariant system with unknown parameters or time-variant system with slow drifting parameters [12, 13]. While the system has abrupt changes in parameters, the algorithm cannot find the exact identification model and will respond slowly to system parameter variations. To solve this kind of problem, MMAC has been a very useful tool in recent years. Since MMAC was presented in 1970s, it has attracted a lot of attention of experts [12–1]. MMAC is an effective approach to solve problems such as time variations and uncertainties. It has the ability to improve the transient responses and the control performance. According to the dynamic character of controlled plant, multiple models are set up to cover the uncertainty of parameter. Much research has been done on continuous-time and discrete-time linear systems [13, 14]. For nonlinear system, only a few results have been given. In recent years, the MMAC based on neural network has been considered by some researchers [14, 15]. But in these papers, the nonlinear system has been modeled by the combination of linear model (the main part) and neural network model (the unmodeled dynamics). The multiple models are still multiple linear models with different parameters, and neural network is used only to compensate for the modeling error of linear model. In this case, the nonlinear system should not be very complex, and too big modeling error between the system and linear model is forbidden. The parameter and structure uncertainty of a relatively complex nonlinear system cannot be modeled by this method. This kind of MMAC with neural network still follows the main ideas of linear MMAC.

In this paper, we consider delay-difference system with multiple delays of Hopfield neural networks of the form

$$u(k+1) = -Cu(k) + \sum_{i=1}^{m} B_i S(u(k-h_i)) + f \,, \tag{1}$$

where $u(k) \in \Omega \subseteq \mathbf{R}^n$ is the neuron state vector, $0 \le h_1 \le \ldots \le h_m$, $C = diag\{a_1, \ldots, a_n\}$, $c_i \ge 0$, $i = 1, 2, ..., n$ is the constant relaxation matrix, $B_i, i = 1, 2, ..., m$ are $n \times n$ constant weight matrices, $f = (f_1, \ldots, f_n) \in \mathbf{R}^n$ is the constant external input vector and $S(z) = [s_1(z_1), \ldots, s_n(z_n)]^T$ with $s_i \in C^1\left[\mathbf{R}, (-1,1)\right]$ where s_i is the neuron activations and monotonically increasing for each $i = 1, 2, ..., n$.

The asymptotic stability of the zero solution of delay-difference system with multiple delays of Hopfield neural networks has been developed during the past several years. Much less is known regarding the asymptotic stability of the zero solution of the control discrete-time system of neural networks. Therefore, the purpose of this paper is to establish sufficient condition for the asymptotic stability of the zero solution of (1) in terms of certain matrix inequalities.

II. PRELIMINARIES

The following notations will be used throughout the paper. $\mathbf{R}^+$ denotes the set of all non-negative real numbers; $\mathbf{Z}^+$ denotes the set of all non-negative integers; $\mathbf{R}^n$ denotes the n-finite-dimensional Euclidean space with the Euclidean norm $\|.\|$ and the scalar product between x and y is defined by $x^T y$; $\mathbf{R}^{n\times m}$ denotes the set of all $(n\times m)$-matrices; and A^T denotes the transpose of the matrix A; Matrix $Q\in\mathbf{R}^{n\times n}$ is positive semidefinite $(Q\geq 0)$ if $x^TQx\geq 0,$ for all $x\in\mathbf{R}^n$. If $x^TQx>0 (x^TQx<0$, resp.) for any $x\neq 0$, then Q is positive (negative, resp.) definite and denoted by $Q>0, (Q<0,$ resp.). It is easy to verify that $Q>0, (Q<0,$ resp.) iff $\exists\beta>0:$ $x^TQx\geq\beta\|x\|^2, \forall x\in\mathbf{R}^n,$

($\exists\beta>0:\ x^TQx\leq-\beta\|x\|^2, \forall x\in\mathbf{R}^n,$ resp.) .

Fact 1 For any positive scalar ε and vectors x and $y,$ the following inequality holds:

$$x^Ty+y^Tx\leq\varepsilon x^Tx+\varepsilon^{-1}y^Ty.$$

Let us denote $V_\delta=\{x\in\mathbf{R}^n:\ \|x\|<\delta\}.$

Lemma 1 The zero solution of difference system is asymptotic stability if there exists a positive definite function $V(x):\mathbf{R}^n\to\mathbf{R}^+$ such that

$$\exists\beta>0:\Delta V(x(k))=V(x(k+1))-V(x(k))\leq-\beta\|x(k)\|^2,$$

along the solution of the system. In case the above condition holds for all $x(k)\in V_\delta$, we say that the zero solution is locally asymptotically stable.

Lemma 2 [7] For any constant symmetric matrix $M\in\mathbf{R}^{n\times n}$, $M=M^T>0$, scalar $s\in\mathbf{Z}^+/\{0\}$, vector function $W:[0,s]\to\mathbf{R}^n$, we have

$$s\sum_{i=0}^{s-1}(w^T(i)Mw(i))\geq\left(\sum_{i=0}^{s-1}w(i)\right)^T M\left(\sum_{i=0}^{s-1}w(i)\right).$$

We present the following technical fact and lemmas, which will be used in the proof of our main result.

III. MAIN RESULTS

In this section, we consider the sufficient condition for asymptotic stability of the zero solution u^* of (1) in terms of certain matrix inequalities. Without loss of generality, we can assume that $u^*=0, S(0)=0$ and f =0 (for otherwise, we let $x=u-u^*$ and define $S(x)=S(x+u^*)-S(u^*)$).

The new form of (1) is now given by

$$u(k+1)=-Cu(k)+\sum_{i=1}^{m}B_iS(u(k-h_i)) \tag{2}$$

Throughout this paper we assume the neuron activations $s_i(x_i)$, $i=1,2,\ldots,n$ is bounded and monotonically nondecreasing on $\mathbf{R}$, and $s_i(x_i)$ is Lipschitz continuous, that is, there exist constant $l_i>0,\ i=1,2,\ldots,n$ such that

$$|s_i(v_1)-s_i(v_2)|\leq l_i|v_1-v_2|,\quad \forall v_1,v_2\in\mathbf{R}. \tag{3}$$

By condition (3), $s_i(x_i)$ satisfy

$$|s_i(x_i)|\leq l_i|x_i|,\quad i=1,2,...,n. \tag{4}$$

Theorem 1 The zero solution of the delay-difference system (2) is asymptotically stable if there exist the symmetric positive definite matrices $P(k)$, $G_i(k)$, $W_i(k)$, $i=1,2,...,m$, and $L=diag[l_1,\ldots,l_n]>0$ satisfying the following matrix inequalities:

$$\psi = \begin{bmatrix} (0,0) & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & (1,1) & (1,2) & \cdots & (1,m) & 0 & 0 & 0 & \cdots & 0 \\ 0 & (2,1) & (2,2) & \cdots & (2,m) & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & (m,1) & (m,2) & \cdots & (m,m) & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & (m+1,m+1) & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & (m+2,m+2) & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & (2m,2m) \end{bmatrix} < 0, \tag{5}$$

where

$$(0,0) = C^T P(k)C - P(k) + \sum_{i=1}^{m}\left(h_i G_i(k) + W_i(k)\right)$$

$$(1,1) = LB_1^T P(k)B_1 L + \varepsilon_1^{-1} LL - W_1(k),$$

$$(1,2) = LB_1^T P(k)B_2 L + \varepsilon_1^{-1} LL,$$

$$(1,m) = LB_1^T P(k)B_m L + \varepsilon_1^{-1} LL,$$

$$(2,1) = LB_2^T P(k)B_1 L + \varepsilon_1^{-1} LL,$$

$$(2,2) = LB_2^T P(k)B_2 L + \varepsilon_1^{-1} LL - W_2(k),$$

$$(2,m) = LB_2^T P(k)B_m L + \varepsilon_1^{-1} LL,$$

$$(m,1) = LB_m^T P(k)B_1 L + \varepsilon_1^{-1} LL,$$

$$(m,2) = LB_m^T P(k)B_2 L + \varepsilon_1^{-1} LL,$$

$$(m,m) = LB_m^T P(k)B_m L + \varepsilon_1^{-1} LL - W_m(k),$$

$$(m+1,m+1) = -h_1 G_1(k),$$

$$(m+2,m+2) = -h_2 G_2(k),$$

$$(2m,2m) = -h_m G_m(k).$$

Proof Consider the Lyapunov function $V(y(k)) = V_1(y(k)) + V_2(y(k)) + V_3(y(k))$, where

$$V_1(y(k)) = x^T(k)P(k)x(k),$$

$$V_2(y(k)) = \sum_{i=1}^{m}\sum_{j=k-h_i+1}^{k}(h-k+i)x^T(j)G_i(k)x(j),$$

$$V_3(y(k)) = \sum_{i=1}^{m}\sum_{j=k-h_i+1}^{k}x^T(j)W_i(k)x(j),$$

$P(k), G_i(k), W_i(k)$, $i = 1,2,...,m$ being symmetric positive definite solutions of (5) and $y(k) = \left[x(k), x(k-h_1), \ldots, x(k-h_m)\right]$.

Then difference of $V(y(k))$ along trajectory of solution of (2) is given by

$\Delta V(y(k)) = \Delta V_1(y(k)) + \Delta V_2(y(k)) + \Delta V_3(y(k))$, where

$$\begin{aligned}\Delta V_1(y(k)) &= V_1(x(k+1)) - V_1(x(k)) \\ &= [-Cx(k) + \sum_{i=1}^{m} B_i S(x(k-h_i))]^T P(k) \\ &\times [-Cx(k) + \sum_{i=1}^{m} B_i S(x(k-h_i))] \\ &-x^T(k)P(k)x(k) \\ &= x^T(k)[CP(k)C - P(k)]x(k) \\ &-\sum_{i=1}^{m} x^T(k)CP(k)B_i S(x(k-h_i)) - \sum_{i=1}^{m} S^T(x(k-h_i))B_i{}^T P(k)Cx(k) \\ &+\sum_{i=1}^{m}\sum_{j=1}^{m} S^T(x(k-h_i))C_i{}^T PC_j S(x(k-h_j)),\end{aligned}$$

$$\Delta V_2(y(k)) = \Delta\left(\sum_{i=1}^{m}\sum_{j=k-h_i+1}^{k}(h_i - k + j)x^T(j)G_i(k)x(j)\right) = \sum_{i=1}^{m} h_i x^T(k)G_i(k)x(k) - \sum_{i=1}^{m}\sum_{j=k-h_i+1}^{k} x^T(j)G_i(k)x(j),$$

$$\Delta V_3(y(k)) = \Delta\left(\sum_{i=1}^{m}\sum_{j=k-h_i+1}^{k} x^T(j)W_i(k)x(j)\right) = \sum_{i=1}^{m} x^T(k)W_i(k)x(k) - \sum_{i=1}^{m} x^T(k-h_i)W_i(k)x(k-h_i). \quad (6)$$

where (4) and **Fact 1** are utilized in (6), respectively.

Note that

$$-\sum_{i=1}^{m} x^T(k)CP(k)B_i S(x(k-h_i)) - \sum_{i=1}^{m} S^T(x(k-h_i))B_i{}^T P(k)Cx(k) \le \varepsilon_1 \sum_{i=1}^{m}\sum_{j=1}^{m} x^T(k)CP(k)B_i B_j{}^T P(k)Cx(k)$$

$$+\varepsilon_1^{-1}\sum_{i=1}^{m}\sum_{j=1}^{m} S^T(x(k-h_i))S(x(k-h_j)),$$

$$\sum_{i=1}^{m}\sum_{j=1}^{m} S^T(x(k-h_i))B_i{}^T P(k)B_j S(x(k-h_j)) \le \sum_{i=1}^{m}\sum_{j=1}^{m} x^T(k-h_i)LB_i{}^T P(k)B_j Lx(k-h_j),$$

$$\varepsilon^{-1}S^T(x(k))S(x(k)) \le \varepsilon^{-1}x^T(k)LLx(k),$$

hence

$$\begin{aligned}\Delta V_1 \le\ & x^T(k)[CP(k)C - P(k)]x(k) + \varepsilon_1 \sum_{i=1}^{m}\sum_{j=1}^{m} x^T(k)CP(k)B_i B_j{}^T P(k)Cx(k) \\ &+\sum_{i=1}^{m}\sum_{j=1}^{m} x^T(k-h_i)LB_i{}^T P(k)B_j Lx(k-h_j) \\ &+\varepsilon_1^{-1}\sum_{i=1}^{m}\sum_{j=1}^{m} x^T(k-h_i)LLx(k-h_j) + \varepsilon^{-1}x^T(k)LLx(k),\end{aligned}$$

Then we have

$$\Delta V \le x^T(k)[CP(k)C - P(k) + \sum_{i=1}^{m}(h_i G_i(k) + W_i(k))$$
$$+\varepsilon_1 \sum_{i=1}^{m}\sum_{j=1}^{m} CP(k)B_i B_j{}^T P(k)C]x(k)$$
$$+\sum_{i=1}^{m}\sum_{j=1}^{m} x^T(k-h_i)[LB_i{}^T P(k)B_j L + \varepsilon_1^{-1} LL]x(k-h_j)$$
$$-\sum_{i=1}^{m} x^T(k-h_i)W_i(k)x(k-h_i) - \sum_{i=1}^{m}\sum_{j=k-h_i+1}^{k} x^T(j)G_i(k)x(j).$$

Using **Lemma 2**, we obtain

$$\sum_{i=k-h+1}^{k} x^T(i)G_i(k)x(i) \ge \left(\frac{1}{h}\sum_{i=k-h+1}^{k} x(i)\right)^T (hG_i(k))\left(\frac{1}{h}\sum_{i=k-h+1}^{k} x(i)\right).$$

From the above inequality it follows that:

$$\Delta V \le x^T(k)[CP(k)C - P + \sum_{i=1}^{m} h_i G_i(k) + W_i(k)$$
$$+\varepsilon_1 \sum_{i=1}^{m}\sum_{j=1}^{m} CP(k)B_i B_j{}^T P(k)C + \varepsilon^{-1} LL]x(k)$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{m} x^T(k-h_i)[LB_i{}^T P(k)B_j L + \varepsilon_1^{-1} LL]x(k-h_j)$$
$$-\sum_{i=1}^{m} x^T(k-h_i)W_i(k)x(k-h_i) - \sum_{i=1}^{m}\left(\frac{1}{h_i}\sum_{j=k-h_i+1}^{k} x(j)\right)^T (h_i G_i(k))\left(\frac{1}{h_i}\sum_{j=k-h_i+1}^{k} x(j)\right)$$

$$= \left(x^T(k), x^T(k-h_1), x^T(k-h_2), \ldots, x^T(k-h_m), (\frac{1}{h_1}\sum_{i=k-h_1+1}^{k} x(j))^T, (\frac{1}{h_2}\sum_{i=k-h_2+1}^{k} x(j))^T, \ldots, (\frac{1}{h_m}\sum_{i=k-h_m+1}^{k} x(j))^T\right)$$

$$\times \begin{bmatrix}
(0,0) & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & (1,1) & (1,2) & \cdots & (1,m) & 0 & 0 & 0 & \cdots & 0 \\
0 & (2,1) & (2,2) & \cdots & (2,m) & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & (m,1) & (m,2) & \cdots & (m,m) & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & \cdots & 0 & (m+1,m+1) & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & (m+2,m+2) & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & \ddots & \ddots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & \vdots & \ddots & \ddots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & (2m,2m)
\end{bmatrix}$$

$$\times \begin{pmatrix} x(k) \\ x(k-h_1) \\ x(k-h_2) \\ \vdots \\ x(k-h_m) \\ \left(\frac{1}{h_1}\sum_{j=k-h_1}^{k-1} x(j)\right) \\ \left(\frac{1}{h_2}\sum_{j=k-h_2}^{k-1} x(j)\right) \\ \vdots \\ \left(\frac{1}{h_m}\sum_{j=k-h_m}^{k-1} x(j)\right) \end{pmatrix}.$$

and $$y(k) = \begin{pmatrix} x(k) \\ x(k-h_1) \\ x(k-h_2) \\ \vdots \\ x(k-h_m) \\ \left(\frac{1}{h_1}\sum_{j=k-h_1+1}^{k} x(j)\right) \\ \left(\frac{1}{h_2}\sum_{j=k-h_2+1}^{k} x(j)\right) \\ \vdots \\ \left(\frac{1}{h_m}\sum_{j=k-h_m+1}^{k} x(j)\right) \end{pmatrix}.$$

$$= y^T(k)\psi\, y(k),$$

where

$$(0,0) = CP(k)C - P(k) + \sum_{i=1}^{m} h_i G_i(k) + W_i(k)$$
$$+\varepsilon_1 \sum_{i=1}^{m}\sum_{j=1}^{m} CP(k)B_i B_j^T P(k)C,$$
$$(1,1) = LB_1^T P(k)B_1 L + \varepsilon_1^{-1} LL - W_1(k),$$
$$(1,2) = LB_1^T P(k)B_2 L + \varepsilon_1^{-1} LL,$$
$$(1,m) = LB_1^T P(k)B_m L + \varepsilon_1^{-1} LL,$$
$$(2,1) = LB_2^T P(k)B_1 L + \varepsilon_1^{-1} LL,$$

$$(2,2) = LB_2^T P(k)B_2 L + \varepsilon_1^{-1} LL - W_2(k),$$
$$(2,m) = LB_2^T P(k)B_m L + \varepsilon_1^{-1} LL,$$
$$(m,1) = LB_m^T P(k)B_1 L + \varepsilon_1^{-1} LL,$$
$$(m,2) = LB_m^T P(k)B_2 L + \varepsilon_1^{-1} LL,$$

$$(m,m) = LB_m^T P(k)B_m L + \varepsilon_1^{-1} LL - W_m(k),$$
$$(m+1,m+1) = -h_1 G_1(k),$$
$$(m+2,m+2) = -h_2 G_2(k),$$
$$(2m,2m) = -h_m G_m(k),$$

By the condition (5), ΔV is negative definite, namely there is a number $\beta > 0$ such that $\Delta V(y(k)) \le -\beta \|y(k)\|^2$, and hence, the asymptotic stability of the system immediately follows from **Lemma 1**. This completes the proof.

Remark 1 Theorem 1 gives a sufficient condition for the asymptotic stability of delay-difference system (2) via matrix inequalities. These conditions are described in terms of certain diagonal matrix inequalities, which can be realized by using the linear matrix inequality algorithm proposed in [4]. But Hu and Wang [9] these conditions are described in terms of certain symmetric matrix inequalities, which can be realized by using the Schur complement lemma and linear matrix inequality algorithm proposed in [4].

CONCLUSIONS

This paper was dedicated to the delay-dependent stability of delay-difference system with multiple delays of Hopfield neural networks. A less conservative LMI-based globally stability criterion is obtained with quadratic Lyapunov functional approach and free-weighting matrix approach for periodic delay-difference system with multiple delays of cellular neural networks. One example illustrates the exactness of the proposed criterion.

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