FUZZY LINEAR PROGRAMMING APPROACH FOR DETERMINING THE PRODUCTION AMOUNTS IN FOOD INDUSTRY

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Abstract— In recent years, rapid and correct decision making is crucial for both people and enterprises. However, uncertainty makes decision-making difficult. Fuzzy logic is used for coping with this situation. Thus, fuzzy linear programming models are developed in order to handle uncertainty in objective function and the constraints. In this study, a problem of a factory in food industry is investigated, required data is obtained and the problem is figured out as a fuzzy linear programming model. The model is solved using Zimmerman approach which is one of the approaches for fuzzy linear programming. As a result, the solution gives the amount of production for each product type in order to gain maximum profit.

Index Terms— Food Industry, Fuzzy Linear Programming, Fuzzy Logic, Linear Programming.

I. INTRODUCTION

Decision Making (DM) helps executives for determining the best alternative in any decision problem which comprise of procedures and criteria. DM generally depend on Decision Support Systems (DSS) tools [1].

Fuzzy set theory can be optimum approach to cope with uncertainties in decision problems modelled with linear programming. [2] There are different techniques in fuzzy linear programming. A common characteristic of these techniques is the preliminary defuzzification of the fuzzy variables [1].

Zadeh and Bellman propounded the notion of maximizing the decision for decision making problems. Tanaka et al adjusted it for mathematical programming problems. A fuzzy approach concerning multi-objective linear programming problems was introduced by Zimmermann. Also, Lai and Hwang, Tong Shaocheng, Buckley, among others, deal with that whole parameters are fuzzy. Studies in recent years suggest new techniques with the purpose of ranking fuzzy numbers and coming to an optimal solution [3].

II. FUZZY SETS

Fuzzy set theory is based on the logic underpinned the types of reasoning, which are ambiguous instead of precise [6]. In real life, classical logic is insufficient for many situations and events. When a person ask what the weather is, the answer can be very cold, cold, warm, hot, very hot and extremely hot. Style of thinking and data processing of people can process the uncertain data. However, traditional computer technology cannot overcome this type of data [4].
In fuzzy logic, a proposition can be either true or false. Moreover, it can be also partially true and partially false. That is, an element can has two membership in different sets simultaneously. In this case, trueness degree of the proposition range from zero to one. On the other hand, it also has '1-trueness degree' as degree of falsity [4].

System modelling is used for symbolizing a real physical system. While this process perform, mathematical formula or equation is used. Most of physical systems include uncertainty and they are too sophisticated to model it completely. This type of systems can model with fuzzy logic approximately. Thus, fuzzy system modelling play an important role to describe complicated real world systems [5].

III. FUZZY LINEAR PROGRAMMING

A. Linear Programming

A Linear Programming (LP) problem is a special case of Mathematical Programming problem. From an analytical perspective, a mathematical program attempts to identify an extreme (minimum or maximum) point of a function, which furthermore satisfies a set of constraints. Linear programming is the objective function and the problem constraints are linear [6]. "Float over text" should *not* be selected.
A classical model of LP, also called a crisp LP model, may have the following formulation:

$$\begin{aligned} &\text{Max} \quad Cx \\ &\text{s.t.} \qquad\qquad\qquad (1) \\ &A_i x \le b_i \qquad i-1,\dots,m \end{aligned}$$

in which x is an n × 1 alternative set, C is a 1 × n coefficients of an objective function, A_i is an m × n matrix of coefficients of constraints and b_i is an m × 1 right-hand sides.
The traditional problems of LP are solved with LINDO optimization software and obtain the optimal solution in a precise way. If coefficients of constraints, objective function or the right-hand sides are imprecise, in other words, being fuzzy numbers, traditional algorithms of LP are unsuitable to solve the fuzzy problem and to obtain the optimization.

In the real world, the coefficients are typically imprecise numbers because of insufficient information, for instance, technological coefficients. Many researchers formed Fuzzy Linear Programming of various types, invented approaches to convert them into crisp LP, and finally solved the problems with available software [7].

B. Fuzzy Linear Programming

Fuzzy linear programming (FLP) follows from the fact that classical linear programming is often insufficient in practical situations. In reality, certain coefficients that appear in classical LP problems may not be well-defined, either because their values depend on other parameters or because they cannot be precisely assessed and only qualitative estimates of these coefficients are available. FLP is an extension of classical linear programming and deals with imprecise coefficients by using fuzzy variables [8].

We consider the FLP Problem

$$\begin{aligned} &Max\ \tilde{Z} = \tilde{C}^T x \\ &s.t. \\ &\tilde{A}\, x \lesssim \tilde{b} \\ &x \geq 0 \end{aligned} \qquad (2)$$

The solution of this problem is to find the possibility distribution of the optional objective function Z. Many researchers had handled this problem by converting the fuzzy objective function and the fuzzy constraints into crisp ones [9].

IV. TYPES OF FUZZY LINEAR PROGRAMMING

Fuzzy linear programming model divide into parts in terms of fuzzy coefficients. For instance, while objective function is fuzzy, constraints cannot be fuzzy.

A. Objective Function is Fuzzy

In a real life, there are many situations that parameters of objective function (profit and cost) are imprecise. FLP model of this was propounded by Verdegay.

B. Right-Hand Sides are Fuzzy

There are two approach for this type of problem. While first approach concerning asymmetric models belongs to Verdegay, second approach concerning symmetric model belongs to Werners.

C. Right-Hand Sides and Coefficients of Constrains are Fuzzy

Negoita and Sularia developed an approach for this type of FLP model.

D. Objective Function and Constrains are Fuzzy

As it is understood the title, in this model, both objective function and constrains involve fuzziness. Zimmermann and Chanas have different approaches about it.

F. All Coefficients are Fuzzy

Sometimes, all coefficients can be fuzzy in the problem. Carlsson and Korhonen developed the approach for this.

V. ZIMMERMANN METHOD

A LP with a fuzzy objective function and fuzzy inequalities shown by Zimmermann is indicated as follows: [7]

$$\begin{aligned} &C^T x \gtrsim b_0 \\ &(Ax)_i \lesssim b_i \\ &x \geq 0 \quad i = 1, 2, \ldots, m \end{aligned} \qquad (3)$$

Inequality is a symmetrical model of which the objective function becomes one constraint. To write a general formulation, inequality is converted to a matrix form as [7]:

$$-C^T x \lesssim -b_0 \qquad (4)$$

In which

$$B = \begin{bmatrix} -C \\ A_i \end{bmatrix} \quad b = \begin{bmatrix} -b_0 \\ b_i \end{bmatrix} \qquad (5)$$

The inequalities of constraint signify "be as small as possible or equal" that can be allowed to violate the right-hand side b by extending some value. The degree of violation is represented by membership function as [7]:

$$\mu_0(x) = \begin{cases} 0 & ,if\ Cx \leq b_0 - d_0 \\ 1 - \dfrac{b_0 - Cx}{d_0} & ,if\ b_0 - d_0 \leq Cx \leq b_0 \\ 1 & ,if\ \mathrm{Cx} \leq \mathrm{b}_0 \end{cases} \qquad (6)$$

$$\mu_i(x) = \begin{cases} 0 & ,if\ (Ax_i) \geq b_i + d_i \\ 1 - \dfrac{(Ax)_i - b_i}{d_i} & ,if\ b_i \leq (Ax)_i \leq b_i + d_i \\ 1 & ,if\ (Ax)_i \leq b_i \end{cases} \qquad (7)$$

In which d is a matrix of admissible violation.

This problem can be transformed by introducing the auxiliary variable λ as follows:

$$\begin{aligned} &\mu_0(x) \geq \lambda \\ &\mu_i(x) \geq \lambda \\ &\lambda \in [0,1] \end{aligned} \qquad (8)$$

This problem can be stated as linear programming as follows:

$$\begin{aligned} &Max\ \lambda \\ &s.t. \\ &\mu_0(x) \geq \lambda \\ &\mu_i(x) \geq \lambda \\ &\lambda \in [0,1] \end{aligned} \qquad (9)$$

This problem was shown with membership functions of fuzzy objective function and fuzzy constrains as follows:

$$\begin{aligned} &Max\ \lambda \\ &s.t. \\ &1-\frac{b_0 - Cx}{d_0} \geq \lambda \\ &1-\frac{(Ax)_i - b_i}{d_i} \geq \lambda\ ,\ \forall i \\ &\lambda \in [0,1] \\ &x \geq 0 \end{aligned} \qquad (10)$$

After some simplification, fuzzy linear programming model obtain as follows:

$$\begin{aligned} &Max\ \lambda \\ &s.t. \\ &C^T x - \lambda d_0 \geq b_0 - d_0 \\ &(Ax)_i + \lambda d_i \leq b_i + d_i\ ,\ \forall i \\ &\lambda \subset [0,1] \\ &x \geq 0 \end{aligned} \qquad (11)$$

Table I. Data about the application

Variables									
Variable name	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
Unit profits (TRY per tonne)	1509	1509	503	1509	1006	503	1509	1006	503
Expected demands (tonne per month)	25	100	150	30	150	200	30	250	80
Tolerances for demands (tonne per month)	5	50	40	5	40	50	5	40	20
Labour usage (hour per tonne)	0.333	0.333	0.333	0.283	0.283	0.283	0.3	0.3	0.3
Expected profit (TRY)	1,200,000								
Tolerance for profit (TRY)	200,000								
Monthly production capacity (tonne)	1150								
Monthly labour capacity (hour)	364								

VI. APPLICATION

A. Problem Definition

Data used for the application was obtained a factory in food industry. It produce different product types. Since the expected profit and the demand of the product types are uncertain the problem is built as fuzzy linear programming model in order to determine production amounts per month for each product type for maximizing the profit. Data about the production and its constraints is given in Table I.

B. FLP Model

Problem was modelled as monthly basis. The fuzzy linear programming model of the problem is given below:

$$\begin{aligned} &C^T x = 1509x_1 + 1509x_2 + 503x_3 + 1509x_4 \\ &1006x_5 + 503x_6 + 1509x_7 + 1006x_8 + 503x_9 \\ &b_0 = 1{,}200{,}000 \qquad d_0 = 200{,}000 \\ &b_1 = 25 \qquad d_1 = 5 \\ &b_2 = 100 \qquad d_2 = 50 \\ &b_3 = 150 \qquad d_3 = 40 \\ &b_4 = 30 \qquad d_4 = 5 \\ &b_5 = 150 \qquad d_5 = 40 \\ &b_6 = 200 \qquad d_6 = 50 \\ &b_7 = 30 \qquad d_7 = 5 \\ &b_8 = 250 \qquad d_8 = 40 \\ &b_9 = 80 \qquad d_9 = 20 \end{aligned} \qquad (12)$$

$$\begin{aligned} &Max\ \lambda \\ &s.t. \\ &1509x_1 + 1509x_2 + 503x_3 + 1509x_4 + \\ &1006x_5 + 503x_6 + 1509x_7 + 1006x_8 + \\ &503x_9 - 200{,}000\lambda \geq 1{,}000{,}000 \\ &x_1 + 5\lambda \leq 30 \\ &x_2 + 50\lambda \leq 150 \\ &x_3 + 40\lambda \leq 190 \\ &x_4 + 5\lambda \leq 35 \\ &x_5 + 40\lambda \leq 190 \\ &x_6 + 50\lambda \leq 250 \\ &x_7 + 5\lambda \leq 35 \\ &x_8 + 40\lambda \leq 290 \\ &x_9 + 20\lambda \leq 100 \\ &\lambda \in [0,1] \\ &x_i \geq 0 \\ &i = 1,2,\ldots,9 \end{aligned} \qquad (13)$$

C. Problem Solution

FLP model of the problem has been solved using Lindo optimization software. Results of the solution are given in TableII and TableIII.

As can be seen from the solution, the factory should produce 28.832 tonnes x_1, 138.321tonnes x_2,

180.657tonnes x_3, 33.832tonnes x_4, 180.657tonnes x_5, 177.885tonnes x_6, 33.832tonnes x_7, 280.657tonnes x_8, 95.329tonnes x_9.

Table II. Results of the solution

Variable	Value	Reduced Cost
λ	0.234	0
X_1	28.832	0
X_2	138.321	0
X_3	180.657	0
X_4	33.832	0
X_5	180.657	0
X_6	177.885	0
X_7	33.832	0
X_8	280.657	0
X_9	95.329	0

Table III. Results of the solution

Row	Slack or Surplus	Dual Price
2	0.000000	-0.000003
3	0.000000	0.003292
4	0.000000	0.003292
5	0.000000	0.000000
6	0.000000	0.003292
7	0.000000	0.001646
8	60.436314	0.000000
9	0.000000	0.003292
10	0.000000	0.001646
11	0.000000	0.000000
12	14.192631	0.000000
13	0.000000	0.001646

Total profit of the factory can be calculated as follows:

$$\begin{aligned}&(1509\times 28.832)+(1509\times 138.321)+(503\times 180.657)\\&(1509\times 33.832)+(1006\times 180.657)+(503\times 177.885)\\&(1509\times 33.832)+(1006\times 280.657)+(503\times 95.329)\\&=1046717.83\ TRY\end{aligned} \quad (14)$$

CONCLUSION

In this study, a problem of a factory in food industry was modelled by using fuzzy linear programming. Because the model has fuzziness in both objective function and constraints, it was solved using Zimmerman approach which is one of the approaches for fuzzy linear programming. As a result, the solution gives the amount of production for each product type in order to gain maximum profit.

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