CONTROLLER DESIGN AND INVESTIGATION OF CROSS COUPLING EFFECT IN SISO DIGITAL CONTROLLER FOR HELICOPTER TEST STAND

[1]AMIR MOLAEI, [2]MASOUD RAZBAN, [3]JAVAD DARGAHI

[1,2,3]Department of Mechanical Engineering, Concordia University, Montreal, QC, Canada
E-mail: [1]A_molaei@encs.concordia.ca, [2]m_razban@encs.concordia.ca, [3]dargahi@encs.concordia.ca

Abstract - This paper aims to design a digital controller and investigation of cross coupling effects of the pitch and yaw angle for a two degrees of freedom helicopter test stand. The test stand is comprised of two propellers which orient the pitch and yaw angles of the helicopter. Because of the cross coupling dynamic of the system, motion of the two angles are coupled which has a significant effect on the performance of the controller. In order for the real time implementation of the controller a SISO digital controller based on the root locus techniques is designed to regulate pitch and yaw angles. Implementing the controller, the cross coupling effects of the controller is then investigated for the step input for either of the pitch and yaw angles wherein the obtained results shows that the controller can eliminate the cross coupling effect of the SISO controllers.

Keywords - Controller, Digital Control, Root Locus,

I. INTRODUCTION

Two degree of freedom helicopter test stand is a suitable multi input-multi output (MIMO) platform for investigation of the cross coupling effects of single input-single output (SISO) controllers. A common two DOFs helicopter test stand is comprised of two propeller with perpendicular rotation axis in order to change the pitch and yaw angle of the helicopter. In this study we have chosen the Quanser helicopter test stand, a simplified helicopter model, ideally suited to analyze flight dynamic control schemes. The platform has been used extensively for implementation of various control schemes [1-2] including fuzzy [3-5], adaptive [6-8], robust [9,10], MIMO [11, 12] and observer based [13, 14]. In this paper we investigate digital control method for controller design of yaw and pitch angle of the test stand in order to investigate the cross coupling effect of the controller. Figure 1 shows the experimental setup of the helicopter test stand. It is comprising of two propeller having perpendicular rotation axis with respect to the body of the helicopter. The body itself is hinged with an offset distance with respect to its center of mass on a vertically rotating arm. By rotating the propeller the pitch and yaw angle of the helicopter can be controlled, while having feedback from encoders.

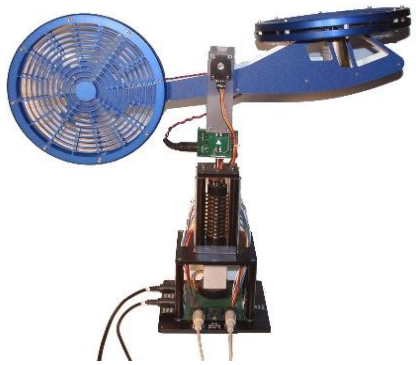


Fig.1. Quanser helicopter test stand

II. MODELLING

A schematic representation of the system is shown in the figure 2. The front propeller controls the elevation of the helicopter nose about the pitch axis θ and the back propeller controls the side to side motions of the helicopter about the yaw axis φ.

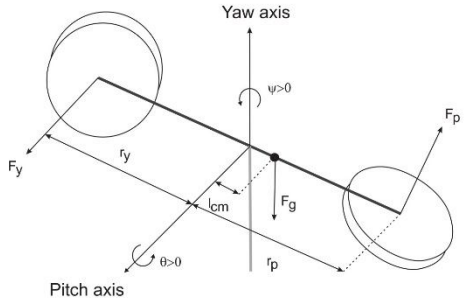


Fig. 2. Representation of the paremeters of the modelling

The transfer function for the pitch and yaw angles are obtained from the catalogue of the manufacturer as below:

$$\theta[s] = \frac{37.2021}{s^2 + 0.2830s + 2.7452} V_p(s) + \frac{3.5306}{s^2 + 0.2830s + 2.7452} V_y(s) \quad (1)$$

$$\varphi[s] = \frac{2.3892}{s^2 + 0.2701s} V_p(s) + \frac{7.461}{s^2 + 0.2701s} V_y(s)$$

III. DISCRETIZING CONTINIOUS TIME SYSTEM

Because of the cross coupling effect, for the design of SISO controller we consider the cross coupling effect as a step disturbance and separately design the

controllers for pitch and yaw angles. Controllers are designed to have a design specification as described in table 1, while eliminating the step disturbance.

From the above specifications we found that:

Table 1: Design specifications of the system response

DSP1	$\exp\left(-\frac{\xi\pi}{\sqrt{1-\xi^2}}\right) \leq 0.2$	$0.5 \leq \xi$
DSP2	$\frac{4.6}{\xi\omega_n} \leq 18$	$0.3 \leq \xi\omega_n$
DSP3	$\frac{1.8}{\omega_n} \leq 3$	$0.6 \leq \omega_n$

Based on the design specification the closed loop system has an equivalent continuous time characteristic equation as:

$$t_r = 3 \rightarrow T = \frac{t_r}{10} = 0.3 \tag{2}$$

And from the settling time we found that:

$$\frac{t_s}{100} < T < \frac{t_s}{5} \rightarrow 0.18 < T < 3.6 \tag{3}$$

Referring to the above equations, 2 and 3, a sampling time of $T = 0.3$ is reasonable for discretizing the continuous time system. In choosing sampling time it is to be noted that small samples time leads to better reaction of the system to disturbance affecting the system but it may results to large control efforts which must be taken into account. Having the sample rate to be 0.3 the discrete transfer function of the system will be:

$$G[z]_{ZOH} = \frac{0.1047z + 0.1019}{z^2 - 1.922z + 0.922} = \frac{0.326(z + 0.973)}{(z-1)(z-0.922)} \tag{4}$$

The frequency response of the digitized system is shown in the Fig 3.

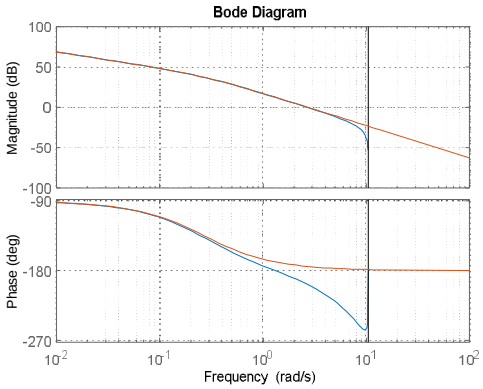


Fig. 3 Frequency response time

With the same sampling time having the sample rate to be 0.3 the discrete transfer function of the system will be for theta:

$$G[z]_{ZOH} = \frac{1.59z + 1.55}{z^2 - 1.68z + 0.918} = \frac{1.59(z + 0.9718)}{z^2 - 1.68z + 0.918} \tag{5}$$

The frequency response of the discrete and continuous system is also shown in the Fig.4:

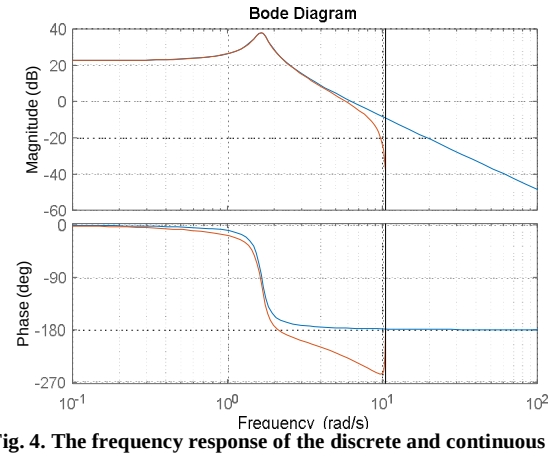


Fig. 4. The frequency response of the discrete and continuous system

For low frequencies the frequency response of the discretized system is approximately similar to that of the continuous time system.

IV. CONTROLLER DESIGN FOR YAW ANGLE

Considering K (z) as the transfer function of the controller, the overall transfer function of the closed loop system will be as:

$$G[s]_{Closed\ Loop} = \frac{G(z).K(z)}{1 + G(z).K(z)} R_\theta(z) + \frac{G(z)}{1 + G(z).K(z)} D_\theta(z) \tag{6}$$

In order to have a zero error with for the disturbance input the controller must be in the form of:

$$K(z) = \frac{\acute{K}(z)}{(z-1)} \tag{7}$$

Having the z transfer function of the system as below we start designing the controller.

$$G[z]_{ZOH} = \frac{0.326(z + 0.973)}{(z-1)(z-0.922)} \tag{8}$$

The general for of the controller will be:

$$K(z) = K\frac{(z+b)}{(z+c)}.\frac{(z-a)}{(z-1)} \tag{9}$$

Choosing a=-0.922 and c=0.0973 we can cancel one stable pole and zero of the system respectively. So the overall transfer function of the controller and system will be:

$$G(z).K(z) = K(z+b)\frac{0.326}{(z-1)^2} \tag{10}$$

We need to find b and K in order to have a suitable performance of the system based on the design specification of the system. Based on the design specification suitable poles for the discrete time

system can be find from that of the continuous time choosing the ξ, and ω_n of the system as 11.

$$\xi = 0.9 \quad \omega_n = 0.6 \tag{11}$$

Being so the final transfer function of the yaw angle controller will be as follow:

$$G[z]_{ZOH} = \frac{0.326(z + 0.973)}{(z-1)(z-0.922)}$$
$$K(z) = 0.933\frac{(z-0.909)}{(z+0.973)}.\frac{(z-0.922)}{(z-1)} \tag{12}$$

Having the controller the step response of the closed loop system will be as Fig. 5:

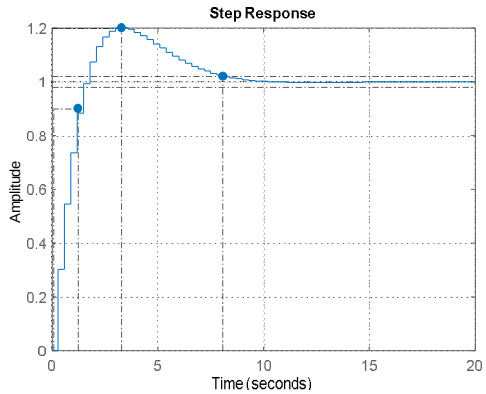


Fig. 5.the controller the step response of the closed loop system

It is clear the response of the system meets the requirements of the problem. Now we find the response of the closed loop system to the step disturbance as Fig. 6:

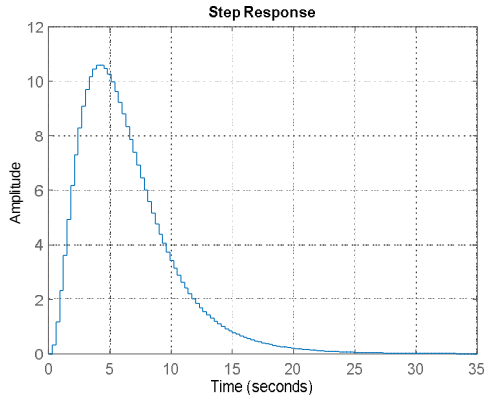


Fig. 6.the response of the closed loop system to the step disturbance

As it is clear controller can compensate the step disturbance but the response to the disturbance is not fast and it takes almost 35 seconds for the disturbance to be completely disappeared.

V. CONTROLLER DESIGN FOR PITCH ANGLE

K (z) is considered as the transfer function of the controller in pitch angle, thus the overall transfer function of the closed loop system is derived as equation 13.

$$G[s]_{Closed\ Loop} = \frac{G(z).K(z)}{1 + G(z).K(z)}R_\theta(z) + \frac{G(z)}{1 + G(z).K(z)}D_\theta(z) \tag{13}$$

A zero error for the system with disturbance input is desired and the controller must be in the form of equation 14.

$$K(z) = \frac{\acute{K}(z)}{(z-1)} \tag{14}$$

By having the transfer function of the system as equation 16 we start designing the controller.

$$G[z]_{ZOH} = \frac{1.59(z + 0.9718)}{z^2 - 1.68z + 0.918} \tag{15}$$

The general form of the controller will be:

$$K(z) = K\frac{(z+a)}{(z+c)}.\frac{(z-a)}{(z-1)} \tag{16}$$

$a = 0.8435 \pm 0.4551i$is selected as the stable poles of the system, hence:

$$G(z).K(z) = K\frac{1.59(z + 0.9718)}{(z+b)(z-1)} \tag{17}$$

In order to have a stable performance of the system, b and K should be find based on the design specification of the system. The discrete time system can be find from that of the continuous time choosing the ξ, and ω_n of the system as 19.

$$\xi = 0.5 \quad \omega_n = 0.6 \tag{18}$$

Following the design procedure, the final transfer function of the yaw angle controller will be shown in equation 20.

$$G[z]_{ZOH} = 0.0094\frac{1.59(z + 0.9718)}{(z-0.82)(z-1)}$$
$$K(z) = 0.0094\frac{z^2 - 1.68z + 0.918}{(z-0.82)(z-1)} \tag{19}$$

Having the controller the step response of the closed loop system will be as Fig.7:

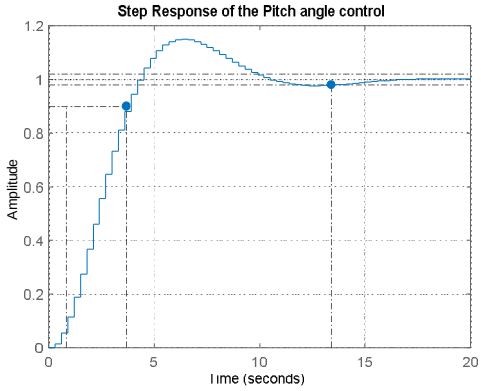


Fig. 7.step response of the closed loop system

It is clear the response of the system meets the requirements of the problem. Now we find the

response of the closed loop system to the step disturbance as Fig. 8:

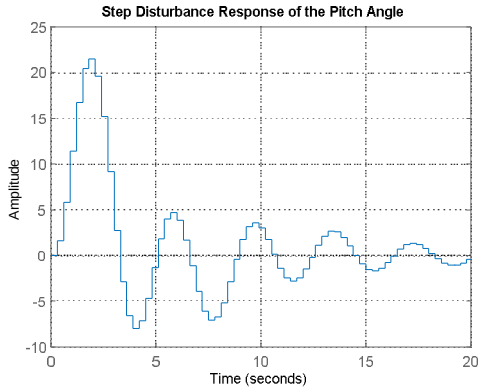


Fig. 8. the response of the closed loop system to the step disturbance

As it is clear controller can compensate the step disturbance but the response to the disturbance but the behavior is oscillating. It is because the dynamic of the system is seen when a disturbance is fed to the system and as the nature of the pitch angle is oscillating we see such a behavior in response to disturbance.

VI. INVESTIGATION OF CROSS COUPLING EFFECT

In order to investigate the cross coupling behavior of the overall system we simulate whole system with the controllers. In order to investigate the cross coupling behavior of the overall system we simulate whole system with the controllers. Transforming the yaw and pitch controller into state space representation and combining them we found the related state space matrix as:

$$A = \begin{bmatrix} 1.82 & -0.82 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0.027 & 0.973 \\ 0 & 0 & 1 & -0.2701 \end{bmatrix}, B = \begin{bmatrix} 0.0625 & 0 \\ 0 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix} \quad (20)$$

$$C = \begin{bmatrix} 0.193 & 0.01407 & 0 & 0 \\ 0 & 0 & -0.833 & 0.837 \end{bmatrix}, D = \begin{bmatrix} 0.009 & 0 \\ 0 & 0.93 \end{bmatrix}$$

For the overall closed loop system we connect the controller and system in series then make the feedback loop wherein the number of state variables are eight. The simulation results is provided in the below figure wherein the step response of the system to either of the inputs and the resulting outputs are shown. Top two figures are for the step response of the pitch angle and the bottom figures are that of the yaw angle. The top left one is the output of the pitch angle and the right one is the resulting yaw angle with the zero input. The bottom right is the output of the yaw angle with step input and the left one is that of the pitch angle with the zero input, Fig.9.

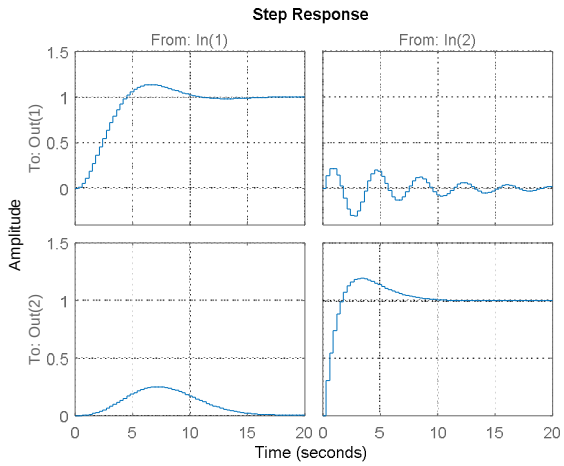


Fig. 9 the response of system for pitch and yaw angles

Referring to the Fig. 10 step input of the pitch angle causes the yaw angle to oscillate with the maximum amount of 0.3 degrees. As it can be seen from the figure below the oscillation is damping which makes the results reasonable.

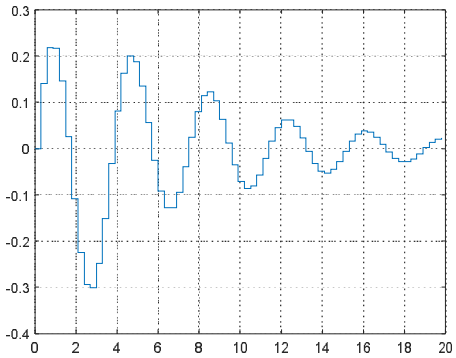


Fig. 10. response for step input of the pitch angle

As it comes from the name, cross coupling also influences the step response of the pitch angle itself with respect to the decoupled one. In order to investigate the effect of cross coupling we find the difference between the step input of the decoupled system for the pitch angle and that of the coupled system. The error between two pitch angles can be find in the Fig. 11.

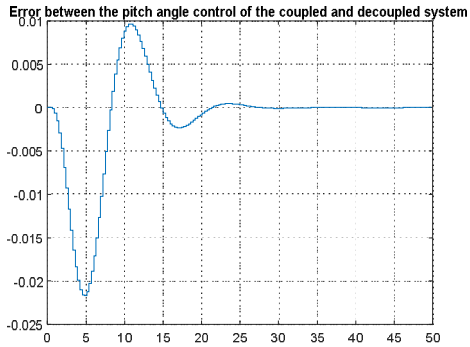


Fig. 11The error between two pitch angles

Referring to the Fig.11 the maximum difference between the two pitch angles is less than 0.025 degrees which is reasonable. The other important

aspect of the error is the decreasing amount of the error which guarantee suitable steady state response. Now we analyze the effect of cross coupling when a step input is feed to the yaw angle. The effects of the yaw angle step input on the pitch angle is found in the Fig.12.

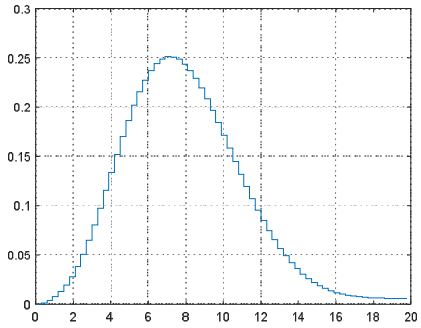


Fig. 12 The effects of the yaw angle step input on the pitch angle

As it is clear the maximum amount of deviation of the pitch angle from zero position is about 0.25 degrees which starts decreasing after the peak making it an acceptable results. In order to investigate the effect of cross coupling on the yaw angle itself we find the difference between the step input of the decoupled system for the yaw angle and that of the cross coupled system. The error between two yaw angles can be find in the Fig. 13.

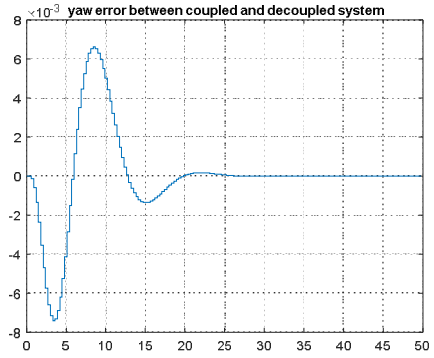


Fig. 13The error between two yaw angles

Referring to the error between two results we found that error is of the order 0.001 which is also decreased making the results acceptable. The decreasing amount of the error guarantees suitable steady state response.

CONCLUSIONS

In this paper the response of the closed loop system to step input is investigated as well as the behavior of the system to the step disturbance. The obtained results meets the requirements of the projects. Then the overall coupled system is simulated considering the cross coupling effects, and the obtained results are discussed in details. The obtained results of the cross coupling simulations shows that the controllers can compensate the cross coupling effects which guarantee a suitable performance of the system.

REFERENCES

[1] Patel, R., Modi, H., Shah, S., Deb, D., "Adaptive Backstepping Control Scheme with Integral Action for Quanser 2-DOF Helicopter",6th International Conference on Advances in Computing, India, 2017

[2] Xin, Y.; Qin, Z.-C.; Wu, W.-G.; Sun, J.-Q., "Partial Model-Free Control of a 2-Input and 2-Output Helicopter System", 9th European Nonlinear Dynamics Conference (ENOC 2017), Tianjin University, Tianjin,2017.

[3] Chang, W., Moon, J.H., Lee, H.J., "Fuzzy Model-Based Output-Tracking Control for 2 Degree-of-Freedom Helicopter",Journal of Electrical Engineering Technology, vol.12, no. 1, 2017.

[4] Khanesar M.A., Kayacan E., "Controlling the Pitch and Yaw Angles of a 2-DOF Helicopter Using Interval Type-2 Fuzzy Neural Networks",Recent Advances in Sliding Modes: From Control to Intelligent Mechatronics,vo;. 24, 2015.

[5] Walker, A., Cohen, K., "Fuzzy Logic Control of Two Two-Degree of Freedom Systems",51st AIAA Aerospace Sciences Meeting,2013.

[6] Subbarao, K., Nuthi, P., Atmeh, G., "Reinforcement learning based computational adaptive optimal control and system identification for linear systems",Annual Reviews in Control,2016.

[7] Nuthi, Pavan and Subbarao, Kamesh, "Experimental Verification of Linear and Adaptive Control Techniques for a Two Degrees-of-Freedom Helicopter",Journal of Dynamic Systems, Measurement, and Control,vol. 137, no. 6, 2015.

[8] Khayati, K., "Multivariable Adaptive Sliding-Mode Observer-Based Control for Mechanical Systems",Canadian Journal of Electrical and Computer Engineering, vol. 38, no. 3, 2015.

[9] Subramanina, R.G., Elumalai, V.K., "Robust MRAC augmented baseline LQR for tracking control of 2 DoF helicopter",Robotics and Autonomous Systems, 2016.

[10] Samadi, B., Rodrigues, L., "A sum of squares approach to backstepping controller synthesis for piecewise affine and polynomial systems",Int. J. Robust. Nonlinear Control,2013.

[11] Perez-D'Arpino, C., Medina-Melendez, W.; Fermin-Leon, L., Bogado, J.M., Torrealba, R.R., Fernandez-Lopez, G., "Generalized Bilateral MIMO Control by States Convergence with Time Delay and Application for the Teleoperation of a 2-DOF Helicopter",2010 IEEE International Conference on Robotics and Automation (ICRA), 2010.

[12] Taskin, Y., "Improving pitch and yaw motion control of twin rotor MIMO system",Journal of Vibroengineering,vol. 16, no. 4, 2014.

[13] Munoz-Vasquez, A.-J., Parra-Vega, V., Sanchez-Orta, A., Romero-Galvan, G., "Finite-time disturbance observer via continuous fractional sliding modes",Transactions of the Institute of Measurement and Control, 2017.

[14] Halsey, K. M., "Analysis and synthesis of nested feedback systems",2005 IEEE Transactions on Automatic Control, vol. 50, no. 7, 2005.

★★★