NOVEL FAULT DIAGNOSIS FOR ROLLER BEARING BY USING MULTISCALE SAMPLE ENTROPY BASED CLUSTERING

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Abstract - The rolling bearing vibration signals are nonlinear and non-stationary when a fault exists. A novel method of rolling bearing fault diagnosis based on multiscale sample entropy (MSE) and Gath-Geva (GG) clustering is proposed for feature extraction. Firstly, the MSE method is used to calculate the sample entropy value in different scales. Secondly, the principal component analysis model is chosen to reduce the dimension of the MSE eigenvector. Then the main components which include the primary fault information are regarded as the input of GG cluster algorithm. The experimental results show that the method proposed in this paper can effectively identify various rolling bearing faults with better performance than fuzzy c-means (FCM) and Gustafson-Kessel (GK) algorithms. In the meantime, the effect of the Gath - Geva algorithm is the best in the three cluster methods.

Index Terms - Fault diagnosis, Gath - Geva cluster algorithm, Multiscale Sample Entropy, Rolling bearing.

I. INTRODUCTION

In the mechanical system, a basic but important component is rolling bearings, whose working performance has great effects on operational efficiency and safety. As the rapid development of computer engineering techniques, many advanced, such as fractal dimension, approximate entropy (AE), sample entropy (SE), support vector machine (SVM), and neural network (NN), synchronous resampling [1] has been widely applied in the fault diagnosis of mechanical systems.

Two key parts of the rolling bearing fault diagnosis, are characteristic information extraction and fault identification. For the information extraction, the rolling bearing's vibration signal is essential. As the vibration signal is unstable, its fault diagnosis is challenging in the mechanical society. Pincus et al. proposed the approximate entropy (AE) method and applied it to fault diagnosis [2], but the results of AE method are sensitive to the length of the data. In certain cases, the value of AE is uniformly lower than the expected one, especially when the data length is short. The AE model was applied to mechanical fault diagnosis by Yan et. al. [3]. In order to overcome this shortcoming, Richman et al. proposed a sample entropy (SE) method to measure the degree of self-similarity and irregularity of time series [4]. Zhu et al. proposed a new method in which empirical mode decomposition (EMD), SE with SVM are combined for rolling bearing fault diagnosis [5]. The vibration signal is decomposed into intrinsic mode functions (IMFs) by EMD. SE is used to calculate the value of IMF which is regarded as the input of SVM model. The SVM method is employed to identify different types of faults. It can be noticed that AE and SE can only reflect the irregularity of time series on a single scale. A method called multiscale entropy (ME) was proposed to measure time sequence irregularity [6], in which the degree of self-similarity and irregularity of time series can be reflected in different scales. For example, the outer race fault and the inner race fault, when rolling bearing running at a particular frequency, can be identified respectively according to the characteristics of the spectrum. The frequencies of vibration signal have deviations when the rolling bearing failure occurs, and the corresponding complexity also has differences. Therefore, multiscale sample entropy (MSE) can be employed as characteristic-index-based fault diagnosis [7]. Based on the MSE, the feature of the vibration signal can be extracted under various conditions, then the eigenvector is regarded as the input of adaptive neuro-fuzzy inference system for rolling bearing fault recognition [8].

Most of the models mentioned above have achieved promising results in rolling bearing fault diagnosis. However, the dimension of eigenvector under multiscale MSE is relatively high. Meanwhile, in most models, such as SVM and NN, label data sets are assumed available. But in practical applications, the data sets are usually unlabeled. Principal component analysis (PCA) method is a commonly used method of dimension reduction, and the characteristics of the original data can be extracted [9]. PCA has been successfully applied in feature extraction in mechanical equipment fault diagnosis

[10]. Fuzzy c-means (FCM) algorithm is a common method for rolling bearing fault diagnosis when the data is unlabeled [11]. FCM algorithm is suitable for a data structure with the homogenous structure, but it can only handle the spherical distance data of the standard specification. Gustafson-Kessel (GK) clustering algorithm is an improved FCM algorithm, in which adaptive distance norm and covariance matrix are introduced. As GK can handle subspace dispersion and scatter along any direction of the data [12], GK cluster has been successfully used in fault diagnosis [13]. As the Euclidean distance is used to compute the distance between two sampleof FCM and GK algorithms, hence they only handle data with a sphere-like structure. Since the distribution patterns of the data are seldom spheres, Gath-Geva (GG) clustering algorithm is proposed for this purpose [14]. Fuzzy maximum likelihood estimation of distance norm can reflect the different shape and orientations of data structure [15].

In summary, a novel method based on MSE, PCA, and GG for rolling bearings fault diagnosis is proposed in this paper. The rest of the paper is organized as follows: A review of MSE and GG method is given in Section 2. Experimental data sources and parameter selection are described, and a proposed method is illustrated in Section 3. A detailed analysis of the experimental results is conducted in Section 4, and conclusions are given in Section 5.

II. RELATED WORK

A. Multiscale Sample Entropy(MSE)

(1)For a time series $X_q=\{x_1,x_2,\cdots x_M,1\le i\le M,1\le q\le N\}$ with length M, where N denotes the number of the sample, m represents embedding dimension in MSE method. The m-dimensional vector $X_m^q(i)$ is formed as:

$$X_q^m(i)=\{x(i),x(i+1),\cdots,x(i+m-1)\},1\le i\le M-m+1 \quad (1)$$

Then a new vector series is obtained.

(2) The maximum absolute distance between two such vectors is defined as:

$$d_{ij}\left[X_q^m(i),X_q^m(j)\right]=\max_{k\in[1,m-1]}\left[|x(i+k)-x(j+k)|\right]$$

$$i,j=1,2,....M-m+1,\ i\ne j \quad (2)$$

(3)Calculating the number of $d_{ij}\left[X_q^m(i),X_q^m(j)\right]\le r$ for each $X_q^m(i)$ to the total number of vectors $M-m+1$. Let A_i^m be the number of the vector that satisfies.

$$A_i^m(r)=\frac{1}{M-m-1}\left(the\ number\ of\ d_{ij}\left[X_q^m(i),X_q^m(j)\right]\le r\right)$$

$$i,j=1,2,....M-m+1,\ i\ne j \quad (3)$$

(4)The average value of the $A_i^m(r)$ is designated as

$$B^m(r)=\frac{1}{M-m}\sum_{i=1}^{M-m}B_i^m(r) \quad (4)$$

Where r represents the boundary width of the exponential function.

(5) Increasing dimension m to $m+1$, then repeat steps (1)-(4) to calculate the value of $B^{m+1}(r)$. Finally, SE is defined as follows:

$$SE(m,r,M)=\sum_{M\to\infty}-ln\frac{B^{m+1}(r)}{B^m(r)} \quad (5)$$

when N is finite, the SE can be estimated by:

$$SE(m,r,M)=lnB^m(r)-lnB^{m+1}(r) \quad (6)$$

SE determines the time sequence irregularity on the single scale. The smaller the value of SE is, the higher time sequence self-similarity can be achieved, and vice versus. ME is described as the time series in different scales and obtained by the coarse-grained process. ME reflects the irregularity and self-similarity of the time series in different scales. The MSE calculation process is as follows:

(1) Given a discrete time series $X_q=\{x_1,x_2,\cdots x_M,1\le i\le M,1\le q\le N\}$, the value of embedding dimension m and similarity tolerance r of the SE are set in advance. A new vector time series called coarse-grained vector $\{y_\tau\}$ is constructed in sequence.

$$y_\tau=\frac{1}{\tau}\sum_{i=(j-1)\tau+1}^{j\tau}X_q,1\le j\le M/\tau \quad (7)$$

Here τ is the scale factor. Especially when τ =1, the coarse-grained time series is the original time series X_q. Thus the original time series is decomposed into a coarse-grained vector series $\{y_\tau\}$, when τ is a non-zero number, the number and length of coarse-grained time series $\{y_\tau\}$ are τ and M/τ respectively.

(2) The SE values are calculated according to different scales factor τ, herer in SE generally takes the standard deviation (SD) of the original time series when calculating the value of SE. This procedure is called MSE analysis.

B. Gath - Geva (GG) clustering algorithm

GG method divides the data set $X_q=\{X_1,X_2,\cdots,X_q,\cdots,X_N\}\ \ 1\le q\le N$ with N samples into c class $(1\le c\le N)$, Each sample has k in Eq. (10) characteristic indexes, so $X_q=\{x_{q1},x_{q2},\cdots,x_{qk}\}$. The membership classification matrix $U=(u_{iq})_{c\times N}$. Here $i=1,2,\cdots,c$ and $q=1,2,\cdots,N$. Elements of u_{iq} means that the

qth sample is classified objects membership belonging to the i[th] class. Meanwhile $u_{iq} \in [0,1]$, $\sum_{i=1}^{c} u_{iq} = 1, 1 \le q \le N$ and $0 < \sum_{i=1}^{c}\sum_{q=1}^{N} u_{iq} < N$. The calculation steps of GG method are as follows:

(1) Setting the membership classification matrix $U = (u_{iq})_{c \times N}$ and the termination tolerance $\varepsilon > 0$

(2) For l = 1, 2, . . .

The cluster centers are calculated by

$$v_i^l = \sum_{q=1}^{N} \left(\mu_{iq}^{l-1}\right)^2 X_q \Big/ \sum_{q=1}^{N} \left(\mu_{iq}^{l-1}\right)^2, 1 \le i \le c, 1 \le q \le N \quad (8)$$

where v_i is the ith cluster center of the matrix $V = (v_1, v_1, \cdots v_i \cdots, v_c)\ 1 \le i \le c$.

The distance measure D_{iq}^2 is computed. The distance of the prototype is calculated based on the fuzzy covariance matrices F_i^l of the cluster.

$$F_i^l = \frac{\sum_{q=1}^{N} \left(\mu_{iq}^{l-1}\right)^2 \left(X_q - v_i^l\right)^T \left(X_q - v_i^l\right)}{\sum_{q=1}^{N} \left(\mu_{iq}^{l-1}\right)^2} \quad 1 \le i \le c \quad (9)$$

(3) The distance function is chosen as:

$$D_{iq}^2\left(X_q, v_i\right) = \frac{(2\pi)^{\frac{n}{2}} \sqrt{\det(F_i)}}{\alpha_i} \exp\left(\frac{1}{2}\left(X_q - v_i^l\right)^T F_i^{-1}\left(X_q - v_i^l\right)\right) \quad (10)$$

Here α_i is the membership average value of the ith cluster $\alpha_i = \frac{1}{N}\sum_{q=1}^{N} \mu_{iq}$

(4)Update the partition matrix.

$$\mu_{iq}^{(l)} = \frac{1}{\sum_{j=1}^{c} \left(D_{ikA_i}\left(X_q, v_i\right) / D_{iq}\left(X_q, v_j\right)\right)^2} \quad (11)$$

Until $\left\| U^{(l)} - U^{(l-1)} \right\| < \varepsilon$, where ε is the termination tolerance of the clustering method.

C. Clustering effect evaluation

The two indicators PC and CE are used to assess the quality of clustering results [16].

Partition Coefficient (PC): it is defined as follows:

$$PC = \frac{1}{N}\sum_{i=1}^{c}\sum_{q=1}^{N} \left(\mu_{iq}\right)^2 \quad (12)$$

where μ_{iq} is the membership value of the sample in ithcluster. The disadvantage of PC is a lack of direct connection with some property of the data themselves. The optimal number of cluster is at the maximum value.

Classification Entropy (CE): it measures the fuzziness of the cluster partition only, which is similar to the PC.

$$CE = -\frac{1}{N}\sum_{i=1}^{c}\sum_{q=1}^{N} \mu_{iq} \log\left(\mu_{iq}\right) \quad (13)$$

When the PC value is close to 1 indicates that the effect of clustering is better, conversely, the CE value is close to 0 indicates that the effect of clustering isbetter [16].

III. EXPERIMENTAL DATA SOURCES AND PARAMETER SELECTION

A. Rolling bearing data set

The experimental data comes from Case Western Reserve University Bearing Data [17]. Single point faults include 7 mils, 14 mils, 21 mils, 28 mils, and 40 mils of fault diameters (1 mils=0.001 inches)were induced to the experimental bearings using electro-discharge machining. Table 1 shows the different working conditions which are under consideration in this study. In Table 1, “NR” denotes the bearings have no faults and “BF”, “IRF” and “ORF” denote fault in a ball, inner race fault, and outer race fault. 1hp (0.18mm), 2hp (0.36mm) and 3hp (0.54mm) are the fault diameters. Meanwhile, each fault type have50 samples with 2048 points.

Fault category	Size(mm)	Motor Speed (rpm)	Number of samples
NR	0	1797	50
IRF	0.1778	1797	50
BF	0.1778	1797	50
ORF	0.1778	1797	50

Table. 1: The rolling bearing experimental data under four bearing conditions.

B. Parameter selection

The performance of MSE depends on the parameters, i.e., embedding dimension m and similarity tolerance r. Generally speaking, m is often fixed to 2. The parameter r determines the width of the boundary of the exponential function, a too narrow one will result in salient influence from noise, while a too broad one is supposed to avoid losing information. It is convenient to fix the value of r by 0.1-0.25 multiplied by the standard deviation (SD) of the original data and max scale factor τ in MSE is selected as 20 [8].

Setting the parameter c=4in GG model, where c is the number of clusters. Meanwhile, the value of termination tolerance $\varepsilon = 1e-6$ .Based on the method of MSE, PCA, and GG in this paper, a rolling bearing fault diagnosis method is put forward as follows:

(1)Each rolling bearing vibration signal in Table. 1 is calculated by using MSE method.

(2)The PCA model is used to extract the eigenvector and reduce data dimension. The detailed information is given in [10]. The number of Principal

component(PCs) is fixed according to the Cumulative contribution rate in PCA, it is employed to sort out the various features according to their importance and relationship with faults from high to low. For data visualization, the first two and first three PCs are selected as the input of GG cluster model.

(3)The features are used as the input of FCM, GK, and GG cluster methods for fault diagnosis.

(4)Finally, the PC and CE two indicators are used to assess the quality of clustering results.

The flow chart of the rolling bearing fault diagnosis method is given in Fig. 1.

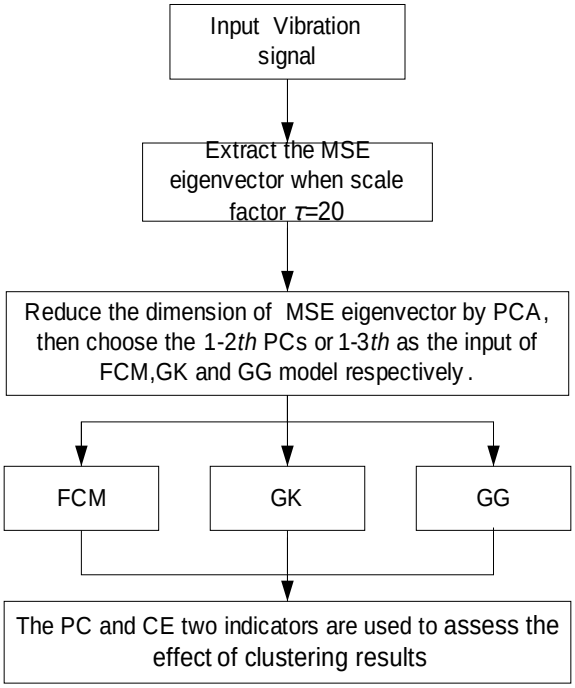


Fig. 1 The flow chart of rolling bearing fault diagnosis method

IV. SIMULATION ANALYSIS OF EXPERIMENT DATA

The data is chosen from the experimental platform equipment in which SKF bearings are used. The approximate motor speed is 1,797 rpm. The data set consists of 200 (N=200) data samples in total, 50 data samples under each fault condition and every data sample have 2048 (M=2048) data points in it. Here with a sample of each state as an example, the time domain waveforms of vibration signals under the four working conditions is shown in Fig. 2.

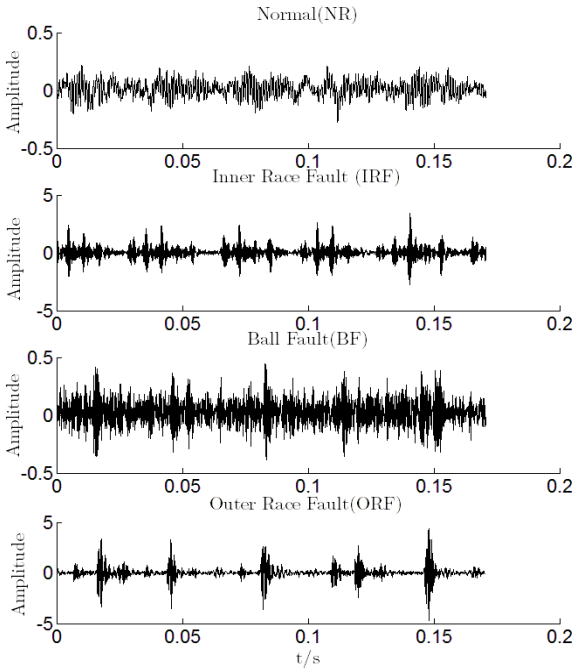
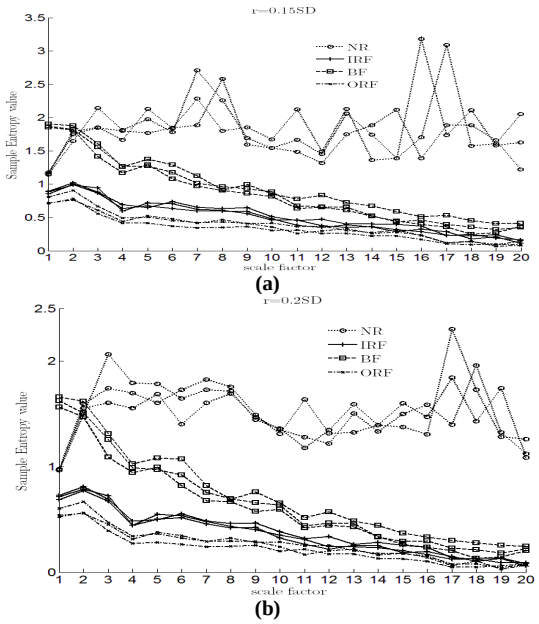
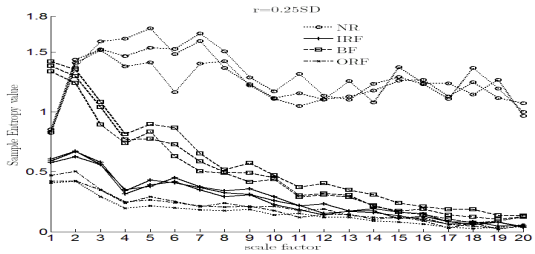


Fig.2. Vibration signals of each bearing condition.

The vertical axis is the acceleration vibration amplitude. Because the influence of noise, it is difficult to find significant differences in different states.It can be seen from Fig. 2 that the amplitude value of the rolling bearing BF signal is the same as NR signal, it is not easy to distinguish between the two signals, and there is no obvious regularity inside the two states.Although IRF and ORF signals have a better vibration regularity,the same amplitude of the signal lead todistinguishing in difficulty.

Setting the embedding dimension m and scale factor τ are 2 and 20. The MSE values of three samples under different conditions when r takes 0.15SD, 0.2SD, and 0.25SD respectively are shown in Fig.3.





(c)

Fig. 3. The MSE values of each sample changes with the scale factor when r takes 0.15SD, 0.2SD, 0.25SD respectively

It can be seen from Fig. 3 that the four states (NR, IRF, OF, BF) are better distinguished when scale factor $\tau \geq 2$. As rolling bearing NR signal with random vibration characteristics causes to its overall MSE values to be maximum,it is consistent with the case of NR signal with the complexity and irregularity in Fig.2. The higher the signal complex, the lower its self-similarity and MSE values. Especially when τ equals one, the coarse-grained time series is the original time series X_q .Concerned with the NR signal without regularity, as the fault vibration signals have fixed periodic shocks in a particular band and obvious vibration periodicity, then the self-similarity and overall MSE values for fault vibration signals are smaller than NR signal. The regularity of BF vibration signal is weaker than the IRF and ORF signals, the self-similarity of the BF signal is the lowest in fault signals, but the corresponding MSE value is the largest. When the outer ring is fixed, the inner ring rotates with the shaft. An IRF signal mechanism is more complex than the ORF signal, and then the self-similarity of the IRF signal is smaller than ORF signal. Therefore, a whole descending sequence of the MSE value is NR>BF>IRF>ORF and consistent with the case in Fig. 3.

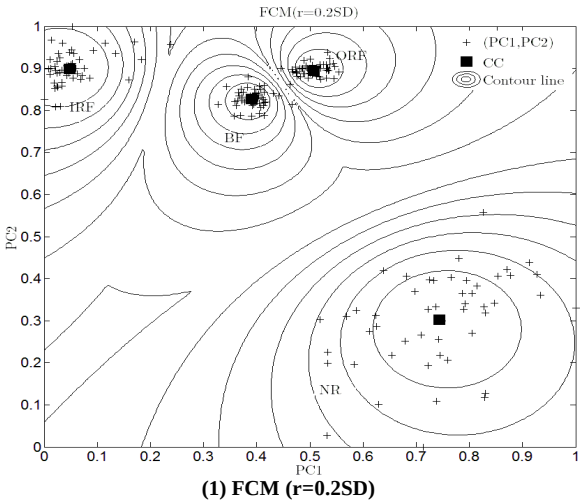
The range of MSE values rolling bearings under different conditions and the overall mean value when r takes 0.15SD, 0.2SD, and 0.25SD are shown in Table. 2. From Table. 2, it can be seen that the overall average value of NR signal is the largest.Their order is NR>BF>IRF>ORF. Then the MSE eigenvectors are regarded as the input of PCA model for data dimension reduction when the parameter rtakes 0.15SD, 0.2SD, and 0.25SD.Then the MSE eigenvectors are regarded as the input of PCA model for data dimension reduction when the parameter r takes 0.15SD, 0.2SD, and 0.25SD.

r	States	Min	Max	Mean
0.15SD	NR	0.9163	3.6721	1.7824
	IRF	0.0984	1.1601	0.5156
	BF	0.0732	2.0156	0.8588
	ORF	0.0368	0.9110	0.3428
0.2SD	NR	0.8215	2.3026	1.4719
	IRF	0.0423	0.9180	0.3692
	BF	0.0269	1.6607	0.6395
	ORF	0.0225	0.6665	0.2344
0.25SD	NR	0.7529	1.8865	1.2541
	IRF	0.0235	0.7428	0.2759
	BF	0.0198	1.4370	0.4889
	ORF	0.0198	0.5060	0.1712

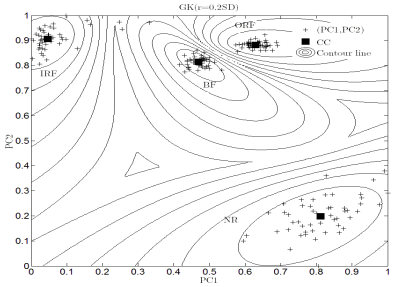
Table.2 The overall average value and the range of rolling bearing MSE values in different conditions when r takes 0.15SD, 0.2SD, and 0.25SD

Then the 1-2nd PCsand1-3th PCs are regarded as the input of FCM, GK, and GG models. Fig. 4 and Fig. 5 are the 2-dimensional and 3-dimensional clustering results of the FCM, GK, and GG models. (As the limited space, we only take r=0.2SD as an example).

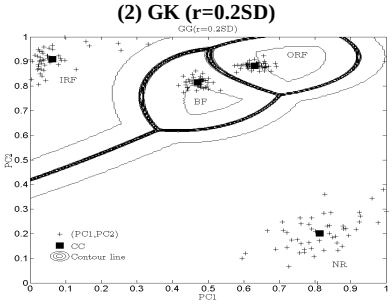
As shown in Fig. 4**,** the contour line of the GG model has no fixed shape.This indicates that the GG model reflects the degree of dispersion in any direction or any subspace of the data.The NR, IRF, BF, and ORF vibration signals at the motor speed take 1797 rpm are effectively distinguished, and the four different states with no overlap by GG clustering method**.**Finally, the two indicators PC and CE are used to assess the quality of clustering results. The result of PC and CE by using FCM, GK, and GG methods are given in Table. 3 As shown in Table. 3, with the gradual increase in the parameter r, the overall PC value increases, and the overall CE value decreases. But the value of PC byusing GG model isthe largest when the dimension takes 2, 3 and r takes 0.15SD, 0.2SD,and 0.25SD respectively, conversely, the corresponding value of CE is the smallest. The meaning of NaN is not a number. GG model is used to calculate CE when the parameter u is a minimal value close to 0, the corresponding values of $log\left(\mu_{ij}\right)$ and $\mu_{ij}\, log\left(\mu_{ij}\right)$ in"(16)" becomes a negative infinity and $0*(-\infty)$, thus the result of CE is not a number but indicates the internal data structure compactness is good.



(1) FCM (r=0.2SD)

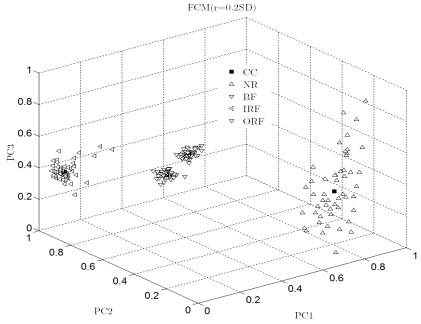


(2) GK (r=0.2SD)

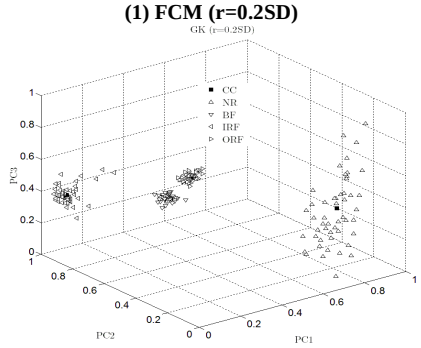


(3) GG (r=0.2SD)

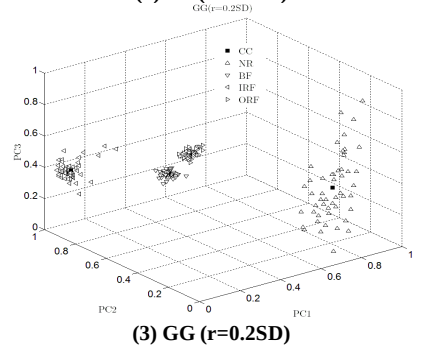
Fig. 4The results of 2-dimensional contour line clustering diagram of FCM, GK, and GG models when r takes 0.15SD, 0.2SD, and 0.25SD



(1) FCM (r=0.2SD)



(2) GK(r=0.2SD)



(3) GG (r=0.2SD)

Fig. 5The results of 3-dimensional contour line clustering results of FCM, GK, and GG models when r takes 0.15SD, 0.2SD,and 0.25SD

Table. 4 The coordinate value of each cluster center point of different vibration signals when k takes 2, 3 and r takes 0.15SD, 0.2SD, and 0.25SD

MODEL	r	FCM		GK		GG	
		PC	CE	PC	CE	PC	CE
	r=0.15 SD	0.8751	0.2688	0.8667	0.2738	0.9990	NaN
MSE-PCA (Dimension=2)	r=0.2 SD	0.9211	0.1824	0.9237	0.1728	1	NaN
	r=0.25 SD	0.9396	0.1446	0.9432	0.1330	1	NaN
	r=0.15 SD	0.8349	0.3452	0.8708	0.2631	0.9996	NaN
MSE-PCA (Dimension=3)	r=0.2 SD	0.8719	0.2813	0.8562	0.2987	1	NaN
	r=0.25 SD	0.8762	0.2758	0.9188	0.1864	1	NaN

Table.3 The value of PC and CE when k takes 2, 3 and r takes 0.15SD, 0.2SD,and 0.25SD

CONCLUSIONS

Based on MSE, PCA and GG methods, a new way of rolling bearings fault diagnosis is presented in this study. The method of MSE is used to decompose the vibration signal into sample entropy values under the multiscale condition and distinguish the different vibration signals in effect. To solve the problem of the MSE eigenvectors dimension is high, the PCA method is selected to reduce its dimension for data visualization. Resulting in a set of PCs. Thus the 1-2 nd PCs and 1-3th PCs are chosen which include the primary fault information as the input of GG clustering method to identify rolling bearing different faults.

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