# LEARNING FROM BAMBOO:OPTIMAL MECHANISMS OF BENDING RESISTANCE FOR CYLINDRICAL STRUCTURES

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**Abstract-** Bamboois a plant with high strength and flexibility. It has a hollow cylindrical shape, which gives it strength over its height, although there is a risk of losing resistance to bending. Here, we examine the optimal form of the bamboo culm in terms of nodes and vascular bundles. A bamboo culm is divided into chambers by nodes and the internode lengthdiffers from bottom to top. Its vascular bundles also have the distinctive feature of a linear distribution. We consider these how characteristics, nodes and vascular bundle distribution, contribute to the suppression of ovalisation caused by bending moments.

**Keywords-** Bamboo, Fiber, Tube Bending, Buckling Mode, Energy Method

## I. INTRODUCTION

Bamboo is widely distributed around the world; there are more than 1250 species, chieflyin temperate, humid climates. It grows so fast that a shoot can mature bamboo in two to three months. It is a plant with high strength and flexibility and these features have made bamboo useful asa structure material and in other applications throughout history. Bamboo is structured with a hollow, cylindrical shape, enabling rapid growth without excess weight, but the shape also reduces its resistance to bending. Therefore, the nodes and linearly distributed vascular bundles are assumed to counteract its susceptibility to bending, which would deform cross section.

The purpose of this study is to examine how the nodes and vascular bundles contribute to bamboo's bending resistanceover many years of evolution.

To investigate its shape properties, we performed a field survey and collected bamboo data. The measurements included internal radius, external radius, wall-thickness, and internode length, from bottom to top. Using these data, we analysed the tendency of internode length, the way bamboo shape is optimizedfor suppressing ovalisation, and the slenderness ratio.

We also investigated these features from considering the linear distribution of vascular bundles. Vascular bundles in bamboo have a linear distribution, which means that the density of the vascular bundles is lowin the inner part of the culm and high in the outer part. The linear distribution of vascular bundles can be modeled by a linear Young's modulus.

Lastly, we obtained a relationship between flattening and the bending moment, using the theorem of minimum strain energy, to examine mathematically how bamboo optimizes its shape.

## II. RELATIONSHIP BETWEEN BENDING AND NODE

### 2.1. Changes in Internode Length

Prior to this research, we performed a field survey to collect samplebamboo measurements. The elements of the data are internal radius, external radius, wall-thickness, and internode length. Figure 1shows the changes in the length of the internodes from the bottom to the top for five bamboo samples.

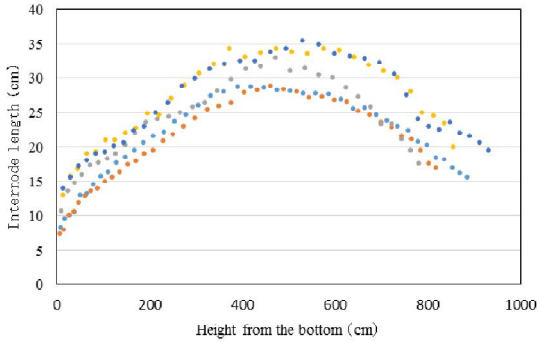



**Fig.1. Changes in internode length**

As shown in Fig.1, the internodes are short at the bottom of a culm, long in the middle, and get shorteragain at the top.Functionally, the bottompart supports the weight of the culm,the middle adds flexibility, and the top partprovides support for branches and leaves by increasing the number of nodes.

### 2.2. Bending Moment of Bamboo

Bamboo can be considered a cantilever, fixed at the bottom, in terms of structural mechanics, which means that the bending moment is large at the bottom and gradually gets smaller at the head. It seems logical that smallerinternode lengths would lead to

larger resistance to the bending. However, the values obtained from the data do not meet this expectation. Therefore, we examined the changes in internode lengthconsidering"end-restraint effects,"Ω, derived by Calladine:

$$\Omega = \sqrt{tL^2 / r^3}\,, \qquad (1)$$

where t is the wall-thickness, L is the length of the internode, and r is the outer radius. Highervalues of Ω lead togreater ease of ovalisation.

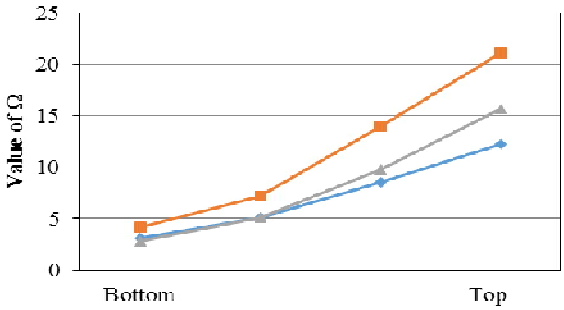



**Fig.2. Changes in end-restraint effect**

From Fig.2,we can see that ovalisation is largely suppressed at the bottom and that the nodes have little influence in suppressing ovalisation at the top. Thus, bamboo has an optimized,hollow cylinderical shape, without excess weight.

### 2.3. Slenderness Ratio of Bamboo

Slenderness ratio is the ratio of culm length to the cross-sectional radius of gyration. Slenderness ratio can be calculated by the following formula:

$$\lambda = l\sqrt{\frac{A}{I}}\,, \qquad (2)$$

where l is the internode length, A is the area of the cross section, and I is the second moment of area. With it, the critical stress for buckling becomes:

$$\sigma_{cr} = \frac{\pi E}{\lambda^2}\,, \qquad (3)$$

whereE is Young's modulus.From these two equations, it is suspected that the larger theslenderness ratio is, the more easily buckling occurs. Figure.3 shows the changes in slenderness ratio from the bottom to the top.It can be seen that slenderness ratio increases monotonically from the bottom to the top, which indicates that the possibility of buckling is much higher in the top than in the bottom part.

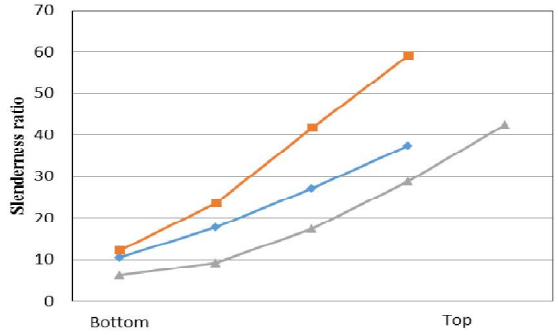



**Fig.3. Changes in slenderness ratio**

## III. RELATIONSHIP BETWEEN VASCULAR BUNDLES AND OVALISATION

### 3.1. Vascular Bundle Distribution in Cross Section of Bamboo

Figure4 shows a cross-sectionalview of bamboo. The upper side is the outer edge and the lower side is the inner edge. As shown in Fig.4, bamboo has a linear distribution of vascular bundles.



**Fig.4. Cross-section of bamboo**

We assumed that this linear distribution of vascular bundles contributes to bamboo's high strength. To quantify this linear distribution, let the coordinate r, whose origin is the center of the tube, and where moving outward along the radius is positive, represent position.Thus, Young's modulus, E, can be described as follows:

$$E_f(r) = b_0 + b_1 r \qquad (a \le r \le a+h)$$

$$b_0 = 13 - b_1 a - \frac{1}{2} b_1 h\,,\ b_1 = \frac{13}{h} \qquad (4)$$

where the wall-thickness is h, and the internal radius is a.

### 3.2.Difference in Longitudinal and Circumferential Modulus

Bamboo has different values for Young's modulus in the longitudinal and circumferential directions. In typical plants, the longitudinal modulus is ten times larger than circumferential one. To evaluate how vascular bundle distribution suppresses ovalisation, we compared the flattening ratio for both a constant and a linear Young's modulus over the cross-section.Here, Young's modulus in the longitudinal direction is $E_1$.

### 3.3. Relationship between Strain Energy and Ovalisation

The total strain energy,in general,is the sum of the strain energy of longitudinal stretching and that of circumferential bending. Thus, the total strain energy per unit length becomes

$$U = U_z + U_\theta\,, \qquad (5)$$

where$U_z$represents the strain energy of longitudinal stretching, and $U_\theta$represents the strain energy of circumferential bending. The bending moment is

$$M = EIC\,. \qquad (6)$$

The strain energy of both directions are difined as

$$U_z = \int MdC = \frac{1}{2}C^2 E_1 I_1 \quad (7)$$

And

$$U_\theta = \int_0^{2\pi} \frac{M^2}{2E_f(r)I_2} rd\theta \quad (8)$$

Here, $I_1$, and $I_2$represent the second moment of cross-sectional area. By the theorem of minimum strain energy, the value for flattening is determined by the condition

$$\frac{\partial U}{\partial \xi} = 0 . \quad (9)$$

When the vascular bundle distribution is constant, the strain energy can be calculated as

$$U_1 = \frac{\pi}{8}\left\{(a+h)^4 - a^4\right\}E_1 C^2\left(1 - \frac{3}{2}\xi + \frac{5}{8}\xi^2\right) + \frac{9\pi}{8}\left\{b_0 + b_1\left(a + \frac{h}{2}\right)\right\}\left\{(a+h)^2 - a^2\right\}\xi^2 \quad (10)$$

andwith alinear distribution of vascular bundles, strain energy is as follows:

$$U_2 = \frac{\pi}{8}\left\{(a+h)^4 - a^4\right\}E_1 C^2\left(1 - \frac{3}{2}\xi + \frac{5}{8}\xi^2\right) + \frac{3\pi}{8}\left\{3b_0\left\{(a+h)^2 - a^2\right\} + 2b_1\left\{(a+h)^3 - a^3\right\}\right\}\xi^2 \quad (11)$$

Figure 5shows the difference between flattening ratiosfor a linear and a constant Young's modulus.$\xi_1$is the flattening ratio when the vascular bundle distribution is constant.$\xi_2$ is the one with a linear distribution of vascular bundles.

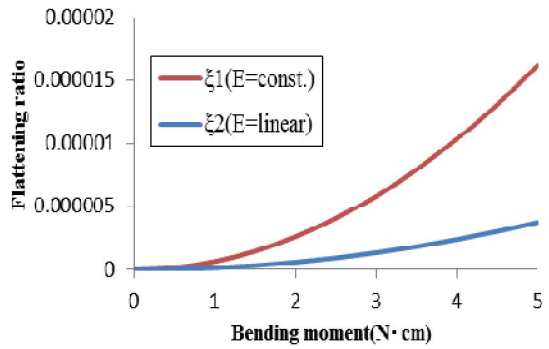



**Fig.5. Flattening ratiochange as a function of bending moment**

We used the representative values for inner radius,a,and wall-thickness,h, from the field survey data; thevalues are a=2.168cm and h=0.632cm.The result indicates that the linear distribution of vascular bundles suppresses ovalisation more efficiently than the constant Young's modulus, which means that bamboo suppresses ovalisation effectively with linearl destirbuted vascular bundles.

## CONCLUSIONS

*Major conclusions of this study are as follows:*

1. Bamboo is an optimized hollow cylinder in terms of suppressing ovalisation. It optimizes the length of internode, radius, and wall-thickness to protect its slender, light bodyfrom deformation. Also, each part of a bamboo has a role as follows: the bottom is for supporting its own weight, the middle extends its flexibility, and the head supports its branches.
2. Bamboo geneates a high resistance to bending by changing its Young's modulus in the cross-section. This linear Young's modulus results from bamboo's linear distribution of vascular bundles; the vascular bundle distribution is dense toward the outer side and sparse toward the inner side of the hollow tube.
3. The linear distribution of vascular bundles in the radial direction enhances the bamboo strength.

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