INFLUENCE OF ELASTICITY OF THE VIBRO-CONVEYER WORKING MEMBER BOTTOM ON THE VIBRATORY DISPLACEMENT OF THE FRIABLE MATERIAL

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Abstract - A vibratory transportation of the friable materials is mostly realized on the rigid working surface and influence of its finite rigidity (elasticity) on the vibratory technologic process of the friable material is not studied sufficiently.A dynamical model of the vibratory system "vibro-exciter – working member with elastic bottom – friable load" and mathematical description in the form of differential equations are considered in the presented report.By mathematical modeling is studied law of the friable material displacement under action of the working member double vibrations – vibrations of the working member frame + proper deformations of the bottom".It is shown that deformations of the frame bottom caused by the frame inertia can be used for increase of both, intensity of displacement and velocity of transportation of the material. Some results of the modeling and a scheme of the working member new construction are presented in the report.

Key words - Vibratory Displacement of the Friable Material, Elastic Working Member, Spatial Vibrations, Mathematical Modeling.

I. INTRODUCTION

One of the important factors having influence on the process of vibratory transportation is an elasticity of the working member that is revealed more or less depending on its design sizes.

A working member is usually considered in the theory of vibratory transportation as a rigid body. Series of researches are carried out on the base of such approach where as a model of the transported material are adopted an individual element (material point) or mass loads friable materials) [3, 4, 5, 6, 7].
The influence of dynamical loads on the elastic elements is studied in the works [8, 9, 10] where axial and plane elastic elements are considered and as loads are adopted unit elements moving separately or in pairs with the help of the conventional elastic elements.
An undulating transportation when the lateral and longitudinal waves are propagated through the elastic tape located on the surface of the rigid plane is considered in work [2] where by variation of their parameters can be obtained running waves of various configurations influencing a character of the transportation process.
Direct influence of the working member finite rigidity on the vibratory transportation process is studied insufficiently [1, 8].

A DYNAMICAL MODEL AND EQUATIONS OF SPATIAL MOVEMENT OF THE SYSTEM "WORKING MEMBER WITH ELASTIC BOTTOM-TECHNOLOGICAL LOAD"

Since a working member of the vibratory machine as well as a load to be displaced perform a spatial movement it is expedient to consider a vibratory movement of the system "vibro-exciter - working member (frame + elastic bottom)" in space [7, 8]. Besides, some spatial (non-working) vibrations in combination with the basic (working) vibrations may turn out to be useful from the standpoint of improvement of the technologic process.
We present a working member in the form of the rectangular plate with corresponding sizes and elastic characteristics (Fig. 1). Coordinate system $O_1x_1y_1z_1$ describes a movement of the working member frame (m_f) together with the elastic bottom (m_1)whose axis makes an angle with the horizon and direction of vibrations coincides with axis O_2x_2. A movement of the technologic load (M_3) whose model is presented by the body having a form of cube, where is located the whole mass of the load [10] is described in the coordinate system $O_3x_3y_3z_3$ connected to the working member.
For describing a rotary movement of mass M a coordinate system $O_3x_3y_3z_3$ is connected to it. Rotary movements of the working member and technologic load are described by Euler's angles $\theta_1, \psi_1, \varphi_1$ and $\theta_2, \psi_2, \varphi_2$.

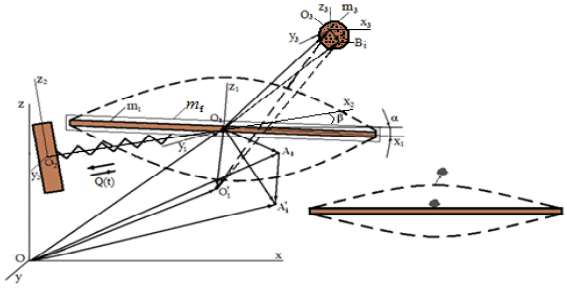


Fig.1. A movement of the loaded working member with elastic bottom in space.

Deformations of the working member elastic bottom are provided for in mathematical model [2, 8] of dynamics of the system "load - working member - vibro-drive" in this case. Instead of consideration separately the active and reactive parts of the vibro-machine is considered the one - reduced mass of the working member.

On the vibro-machine working member are acting exciting force Q_{ex} from vibro-drive, elastic force Q_{el} from the vibro-drive elastic system, N_z - pressure of the load on the working member. Rigidity k of the elastic suspension is taken to be zero.

Due to the working member deformation points $O_1$and A_i will occupy positions $O_1^{'}$ and $A_i^{'}$ (Fig.1). Correspondingly vectors ED, EA_i and OE will be replaced by expressions

$$\vec{O_1^{'}O_3} = \vec{O_1O_3} - \vec{O_1O_1^{'}};$$

$$\vec{O_1^{'}A_i^{'}} = \vec{r_i} + \vec{A_iA_i^{'}}; \vec{OO_1^{'}} = \vec{OO_1} + \vec{O_1O_1^{'}}. \quad (1)$$

Further we obtain that the working member will only be deformed in the direction of the plate (bottom) minimal rigidity. Therefore, the first expression of the system (1) in the coordinates will take the form

$$(O_1^{'}O_3)_x = x_3;\ (O_1^{'}O_3)_y = y_3; \quad (O_1^{'}O_3)_z = \delta - z_3, \quad (2)$$

where $\delta = \delta_0 \sin\frac{i\pi x_1}{a}\sin\frac{i\pi y_1}{b}$ is elastic deformation in the direction of O_3z_3 axis; a and b - lenght and width of the bottom. Similarly can be obtained projections of the second and third expressions

$$(O_1A_i^{'})_x = r_{ix}; \quad (O_1A_i^{'})_y = r_{iy}; \quad (O_1A_i^{'})_z = r_{iz} + \delta;$$

$$(OO_1^{'})_x = x_1;\ (OO_1^{'})_y = y_1;\ (OO_1^{'})_z = z_1 + \delta\cos\alpha, \quad (3)$$

where $\alpha_1 = \alpha + \beta$; α - angle of inclination of the plate relative to the Ox axis; β– angle of vibrations.

Vector expression of the load kinetic energy considering (1), (Fig. 1), will be determined similarly to the succession adopted for conditions of the working member [7].

$$T_1 = \frac{1}{2}\sum_{i=1}^{n_1} M_i[\vec{V_{O_1^{'}}} + \vec{\omega_{O_1}} \times (\vec{O_1O_3} - \vec{O_1O_1^{'}}) + \quad (4)$$

$$+ \vec{\omega_{O_1^{'}}} \times \vec{r_i} + \vec{V_{O_3}} + \vec{\omega_{O_3^{'}}} \times \vec{r_i^{'}}]^2$$

where $M_i$is mass of the load particle B_i, $V_{O_1^{'}}$ - velocity of the point $O_1^{'}$, ω_{O_1} - angular velocity of the point O_1, $\omega_{O_1^{'}}$ - angular velocity of the point $O_1^{'}$, r_i - radius vector of the particle A_i, V_{O_3} - velocity of the point O_3, ω_{O_3} - angular velocity of the point O_3, $r_i^{'}$ - radius vector of the particle B_i.

Expression of the working member kinetic energy considering (1) will have the form

$$T_2 = \frac{1}{2}\sum_{i=1}^{n_1} m_i[\vec{V_{O_1^{'}}} + \vec{\omega_{O_1}} \times (\vec{r_i} + \vec{O_1O_1^{'}})]^2, \quad (5)$$

where m_i is mass of the particle A_i.

For determining potential energy of the vibro-drive linear elastic system and working member we use the expression

$$U = \frac{1}{2}\sum[k_x(\Delta l_{ix})^2 + k_y(\Delta l_{iy})^2 + k_z(\Delta l_{iz})^2], \quad (6)$$

where in contrast to the existent method coefficients of rigidity k_z, k_y, k_z are determined considering bending rigidityof the working member bottom and projections Δl_{ix}, Δl_{iy}, Δl_{iz} - considering (1). Similarly are determined potential energy of the load and dissipative functions of the system.

Projections of the system kinetic energy on the coordinate axes are determined considering expressions (2) and (3) and will be equal to the sum of kinetic energies of two bodies having masses m_1 and m_3 (Fig.1; here we do not give the expanded expression).

Since deformations of the bottom are considered in the direction of the frame vertical displacement the same external forces considering expressions (2) and (3) will enter the differential equation of the bottom movement that were in the equations of vertical movement of the working member as a rigid body [2, 8].

According to expressions (4), (5), (6) and with the use of Lagrange equation of the second order are obtained interconnected differential equations of spatial movement of the vibro-machine elastic bottom and technologic load. Due to their cumbersomeness we do not fully give them here. We only note that in general equations the components with participation of elastic deformations will have the following form: $\ddot{\delta}\psi_1, \dot{\delta}\dot{\varphi}$ and etc. Besides, to spatial equations of the working member will be added equation of the bottom elastic movement considering spatial movements of the working member and material to be transported.

The interconnected equations of movement of the technologic load and elastic bottom presented below are obtained on the base of the following assumptions: 1) Action of the technologic load on the working member is not provided in the equations of movement of the working member; 2) The damping and elastic forces are provided in each equation 0nly according to those coordinates relative to which the movement is considered ; 3) Due to smallness of the rotary movements a movement of the

technologic load is only considered in the three linear directions; 4) The nonlinear components given in the form of products and also the components of inertial character of no greater than second order are provided in the equations of movement of the load considering elasticity of the working surface.

The noted assumptions ensure to obtain the real qualitative and quantitative pictures of the process of vibro-transportation on the elastic working member with sufficient precision.

The equations of spatial movement of the working member frame together with the bottom will take the form:

$$\ddot{x}_1+\frac{c_x}{m}\dot{x}_1+\frac{k_x}{m}x_1=\frac{1}{m}Q_x(t) \quad ;$$

$$\ddot{y}_1+\frac{c_y}{m}\dot{y}_1+\frac{k_y}{m}y_1=\frac{1}{m}Q_y(t);$$

$$\ddot{z}_1+\frac{c_z}{m}\dot{z}_1+\frac{k_z}{m}z_1=\frac{1}{m}Q_z(t) \quad ;$$

$$\ddot{\theta}_1+\frac{c_\theta}{J_\theta}\dot{\theta}_1+\frac{k_\theta}{J_\theta}=\frac{1}{J_\theta}Q_\theta(t);(7)$$

$$\ddot{\psi}_1+\frac{c_\psi}{J_\psi}\dot{\psi}_1+\frac{k_\psi}{J_\psi}\psi_1=\frac{1}{J_\psi}Q_\psi(t) \quad ;$$

$$\ddot{\varphi}_1+\frac{c_\varphi}{J_\varphi}\dot{\varphi}_1+\frac{k_\varphi}{J_\varphi}\varphi_1=\frac{1}{J_\varphi}Q_\varphi(t),$$

where J_θ, J_ψ, J_φ are moments of inertia of mass m relative to axes of the coordinate system $Oxyz$; $Q_x(t)$, $Q_y(t)$, ... , $Q_\varphi(t)$ - - components of the exciting force.

Independent elastic deformations of the working member bottom are described by the equation

$$\ddot{\delta}+p_\delta^2\delta=-\ddot{z}_1\cos\alpha_1-\frac{1}{\rho h\cos\alpha_1}[c_z'z_1+ \quad (8)$$

$$+c_x'x_1+Q_\delta(t)+Q_z(t)]$$

where $p_\delta=[(i^2a^2+j^2b^2)/a^2b^2]\sqrt{D/\rho h}$; $i=1,2,\ldots,n_1$; $j=1,2,\ldots,n_2$; c_x' and c_z' are determined considering rigidities of the vibro-machine elastic system and bottom (plate); $Q_\delta(t)$ - exciting force from the vibro-drive; Q_z - normal pressure of the load on the bottom surface and it will have the form $Q_z=N_z\sin\frac{i\pi x_1}{a}\sin\frac{j\pi y_1}{b}$,(9)

where $N_z=E^*\dot{z}_3-C^*z_3$; coefficients E^* andD^* describe elastic and damping properties of the friable material and they vary depending on the working member vibratory mode (movement of the load with and without losing the contact with the surface). When a friable material moves on the whole surface of the groove (plate) and it has an even thickness, then

$$Q_z=N_z, \quad (10)$$

i.e. the material to be transported is constantly delivered on the working member (plate) which is constantly subjected to dynamical loads.

A spatial movement of the technologic load is described by the system of differential equations

$$\begin{aligned}
&\ddot{x}_3+(\ddot{x}_1-\ddot{\delta}\psi_1\cos\alpha_1-\dot{\delta}\dot{\psi}_1\cos\alpha_1-\\
&\ddot{z}_1\psi_1(\cos\alpha-(\ddot{x}_1\psi_1-\ddot{\delta}\cos\alpha_1+\ddot{z}_1)\sin\alpha+\\
&+\ddot{y}_1\varphi_1-\ddot{\psi}_1\delta-\ddot{\psi}_1z_3-\ddot{\varphi}_1y_3-2\dot{\varphi}_1\dot{y}_3+\\
&+A\dot{x}_1-B\dot{y}_3+B_1x_3=-(N_z-N_y)sign(\dot{x}_3)+\\
&+g\sin\alpha;\\
&\ddot{y}_3+\ddot{y}_1+(\ddot{z}_1\theta_1-\ddot{x}_1\varphi_1+\ddot{\delta}\theta_1\cos\alpha_1+\\
&\dot{\delta}\dot{\theta}_1\cos\alpha_1)\cos\alpha_1+(\ddot{x}_1\theta_1+\ddot{z}_1\varphi_1+\\
&+\ddot{\delta}\varphi_1\cos\alpha_1+\dot{\delta}\dot{\varphi}_1\cos\alpha_1)\sin\alpha- \quad (11)\\
&-\ddot{\theta}_1z_3-2\dot{\theta}_1\dot{z}_3+\ddot{\theta}_1\delta+\dot{\theta}_1\dot{\delta}+\\
&+2\dot{\varphi}_1\dot{x}_3+B\dot{y}_3+B_2y_3=-N_zsign(\dot{y}_3);\\
&\ddot{z}_3+(\ddot{z}_1+\ddot{x}_1\psi_1+\ddot{\delta}\cos\alpha_1)\cos\alpha+(\ddot{x}_1-\\
&-\ddot{z}_1\psi_1-\ddot{\delta}\psi_1\cos\alpha_1-\dot{\delta}\dot{\psi}_1\cos\alpha_1)\sin\alpha-\\
&-\ddot{y}_1\theta_1-\ddot{\theta}_1y_3-2\dot{\theta}_1\dot{y}_3-2\dot{\psi}_1\dot{x}_3+D_1\dot{z}_1+\\
&+E\dot{x}_1+E_1\dot{z}_3-Cz_3=-N_zsign(\dot{z}_z)-g\cos\alpha;
\end{aligned}$$

where A, B, B_1, B_2, D, E, E_1, C are coefficients determining properties of the material to be transported among which B, D, E, E_1, C vary depending on position of the load relative to the working member surface – the load moves together with the working member or moves separately from it having lost the contact [2].

The influence of the working member elasticity is provided in equations (11) by coordinate δ which enters linear or nonlinear components (in the form of products with other coordinates). Among them a dominant influence have the components where accelerations of the elastic deformations (inertial terms) enter linearly, e. g. $\ddot{\delta}\cos\alpha_1\cos\alpha$ in the first equation and $\ddot{\delta}\cos\alpha_1\sin\alpha$ - in the second.

Mathematical modeling of the process of vibro-transportation was carried out on the base of equations (7), (8), (11).

As a research object was chosen a resonant electromagnetic vibro-feeder C-920 with resonant frequency 50 Hz [2].

To study influence of the working member bottom on the process of vibro-transportation was varied its rigidity or sizes. By variation of the rigidity can be obtained various frequencies of the bottom elastic deformations, including in the resonant zones relative to the working member frame vibrations: in the conditions of the basic (50 Hz), sub-harmonic (25 Hz) and super-harmonic (100 Hz) modes. Such approach permits to study influence of the bottom of various rigidities on the process of transportation.

It should be noted that at coincidence of phases of the bottom elastic and frame vibrations the resultant amplitude of the bottom vibrations increases that promotes increase of velocity of the material displacement.

Generation of the nonlinear resonant vibrations in vibro-feeders is stipulated by interaction of the nonlinear electromagnetic and linear or nonlinear elastic forces [8,10].

SOME RESULTS OF THE MATHEMATICAL MODELING

Some graphical dependences obtained as a result of modeling are given below. On the graphs are used the following notations:δ – deformation of the bottom, V – velocity of displacement of the friable material (load);z_3 – vertical displacement of the material; N_z – normal pressure of the material on the bottom surface; ω_δ – frequency of the bottom elastic free vibrations; ω_x – frequency of vibrations of the working member (frame).

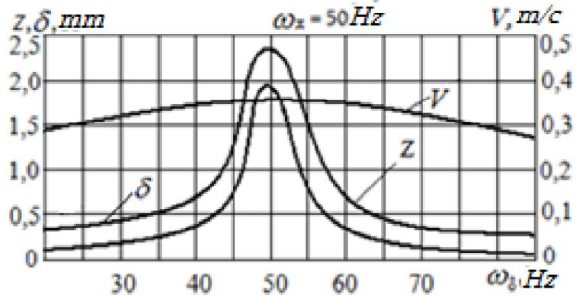


Fig.2. Dependence of V andz_3 on δ at coincidence of the phases ω_δ and ω_x.

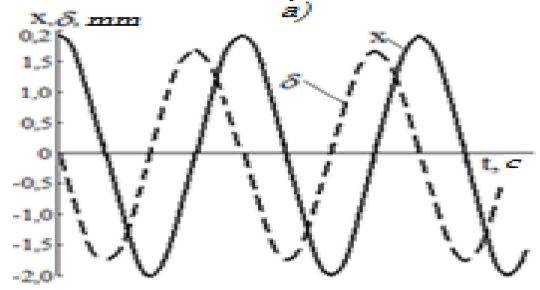


Fig.3. Dependence of V andz_3 on δ at discrepancy between the phases ω_δ and ω_x(Fig.3a).

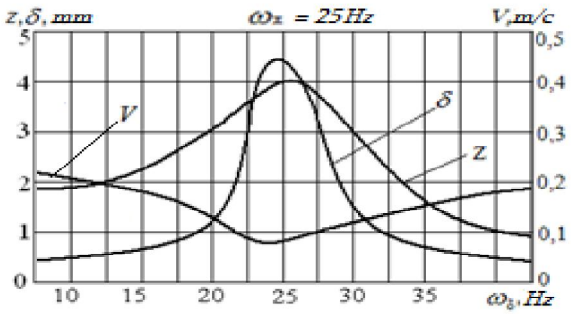


Fig.4. Dependence of V and z_3 onδ at coincidence of the phases ω_δ and ω_x (ω_x=25 Hz).

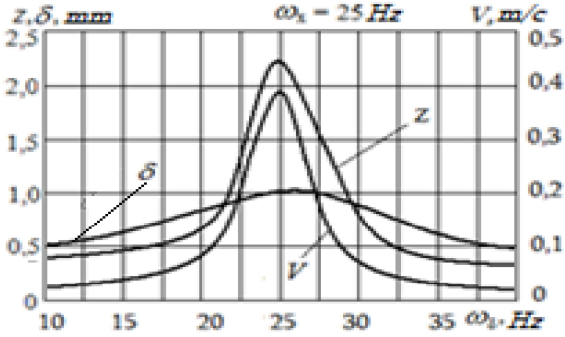


Fig.5. Dependence of V and z_3 on δ at discrepancy between the phases ω_δ and ω_x (ω_x=25 Hz).

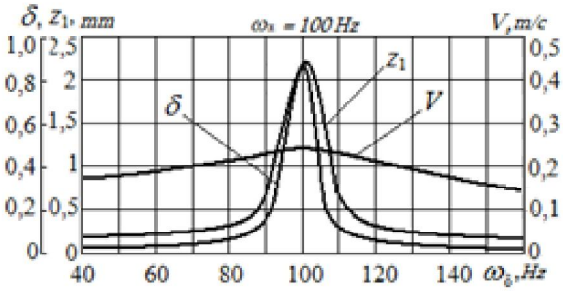


Fig.6. Dependence of V andz_3 on δ at coincidence of the phases ω_δ and ω_x(ω_x=100 Hz).

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CONCLUSIONS

A working member with non-rigid bottom is considered. It should be noted that the deformations enter the other equations too ensuring study of its influence on characteristics of the material movement (velocity, trajectory, etc.).

It is ascertained that elastic vibrations of the bottom enhance the resultant amplitude at coincidence of their phase with the phase of basic (working) vibrations that promotes increase of the material displacement intensity.

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