A HIERARCHICAL WIND DRIVEN OPTIMIZATION METHOD FOR SOLVING THE BI-LEVEL PROGRAMMING PROBLEM

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Abstract - A hierarchical wind driven optimization (WDO) method is proposed to solve the bi-level programming problem. Firstly, the WDO method is used to optimize the lower level programming problem after the upper level variables are initialized, and the positions and velocities of air parcels in the lower level are updated. Then the solutions of the upper level are optimized by the WDO method with the constrained conditions. Finally, the solutions with the minimum sum of sequences of the two-level results are selected as the final solution from the solution set. The experimental results of the proposed algorithm show the effectiveness of the proposed hierarchical wind driven optimization for solving the bi-level programming problems.

Index Terms - Wind Driven Optimization; Bi-level Programming Problem; Constrained Optimization

I. INTRODUCTION

Bi-level programming problem (BLPP) is a hierarchical optimization problem which takes a parameterized optimization problem as its constraint. The general model of BLPP is shown as the equation (1). Both the upper-level and the lower-level programming problems have their own goal functions, as well as constraints, which means that the variables (x, y) to be decided are determined by decision makers from both upper and lower levels respectively. The upper-level decision maker，who makes the decision first，determines the variable x. He or she chooses x according to the objective function of the upper-level programming and its constraints. After that, with x being fixed, the decision maker from the lower-level programming chooses the variable y depending on the objective function and constraints of the lower-level programming.

$$\begin{cases} \min\limits_{x\in X} F(x,y) \\ s.t. G(x,y)\le 0 \\ \min\limits_{y\in Y} f(x,y) \\ s.t. g(x,y)\le 0 \end{cases} \quad (1)$$

In which, x is the upper-level decision variable, and y is the variable to be decided by the lower-level programming，and F, f : $R^n \times R^m \to R$, G(g): $R^n \times R^m \to R^p$ (R^q)，$x \in R^n$, $y \in R^m$.

II. LITERATURE REVIEW

The model of BLPP has been widely used in many fields such as controlling and optimization, economic management programming, optimization of traffic and logistics nets（XU Xiaofeng, ZHANG Wei, LI Ning and XU Huiling, 2015; Aman Gupta and Gerald W.Evans,2009; Vyacheslav V.Kalashnikov,2006; Hyangsook Lee, Yongseok Song and Sangho Choo,2014. BLPP is difficult to solve because it is an NP-hard problem and the non-convexity leads to the complexity of its solution. Many recently research has focused on the solution for BLPP. Based on the quantum-bit's characteristics of chaos and relevance, LI Changbing, DU Maokang and FU Deqiang (2013) proposed a hierarchical chaotic quantum genetic algorithm to figure out the general BLPP, through which a Pareto optimal solution set was obtained. However, the existing solution methods for BLPP are under the condition that both the upper-level and the lower-level objective functions are convex and differentiable or linear. Furthermore, most of these solution algorithms depend on the characteristics of the solution space, thus there is a lack of generality and it is hard to work out the general bi-level programming problems. The Wind Driven Optimization (WDO) algorithm is a kind of heuristic global optimization algorithm based on groups' iteration. Compared to other intelligent algorithms, the WDO algorithm is simpler and easier to implement. Moreover, the number of adjustable parameters is relatively small and it will effectively escape from local optimums. As its updating equations have their actual physical meanings, WDO owns high global searching ability and converges fast. Correspondingly, it is of high efficiency in optimization searching and of strong robustness. On basis of the original WDO algorithm, a hierarchical wind driven optimization algorithm (HWDO) is proposed to solve the BLPP in this paper.

III. BASIC PRINCIPLE OF WDO

The components and structure of the Earth air are very complex, any tiny piece of which (called Air micelle) could be considered as a "point", called an air particle. According to Newton's Second Law of the non-inertial coordinate system, being combined with

the state equation of ideal gas, the WDO algorithm could be acquired by simplifying the model. In the WDO algorithm, the air particles' velocities and positions will change during each iteration to explore the new searching space. Therefore, the variable for velocity can be expressed as $\Delta \boldsymbol{u} = \boldsymbol{u}_{new} - \boldsymbol{u}_{cur}$, in which $\boldsymbol{u}_{cur}$ represents the air particles' velocity in the current iteration, and $\boldsymbol{u}_{new}$ is the air particles' velocity in the next iteration. The updating equation for velocity is expressed as follows:

$$\boldsymbol{u}_{new} = (1-\alpha)\boldsymbol{u}_{cur} - g\boldsymbol{x}_{cur} + \left[RT\,|\frac{1}{i}-1|\,(\boldsymbol{x}_{opt} - \boldsymbol{x}_{cur}) \right] + (\frac{c\boldsymbol{u}_{cur}^{other\,\dim}}{i}) \quad (2)$$

Here, α is the fixed coefficient of friction, whose value usually ranges between 0.8 and 0.9; g is the acceleration of gravity, ranging from 0.6 to 0.7; $\boldsymbol{x}_{opt}$ represents the optimal positions of the air particles; R is the ideal gas constant, while T denotes the temperature, and RT ranges in the interval of [1.0,2.0]; $\boldsymbol{u}_{cur}^{other\,\dim}$ stands for the air particle's velocities in its own dimension. The WDO algorithm is the optimization method based on the air particles' motion, the velocities and positions of which are updated during each iteration. Since the velocities' updating equation is given as equation (2), the updating equation for positions can be easily obtained as follows:

$$\boldsymbol{x}_{new} = \boldsymbol{x}_{cur} + (\boldsymbol{u}_{new}\Delta t) \quad (3)$$

Suppose that the time interval $\Delta t = 1$. For air particles of every dimension, the searching space could be set according to the specific situation，and their updating velocities also change in a certain range, which could be simply set as:

$$u_{new}^{*} = \begin{cases} u_{\max} & \text{if } u_{new} > u_{\max} \\ -u_{\max} & \text{if } u_{new} < -u_{\max} \end{cases} \quad (4)$$

where $u_{\max}$ stands for the maximum velocity.

IV.NUMERICAL EXAMPLE AND DISCUSSION

Use either SI (MKS) or CGS as primary units. (SI units are strongly encouraged.) English units may be used as secondary units (in parentheses). This applies to papers in data storage. For example, write "15 Gb/cm^2 (100 Gb/in^2)." An exception is when English units are used as identifiers in trade, such as "3½ in disk drive." Avoid combining SI and CGS units, such as current in amperes and magnetic field in oversteps. This often leads to confusion because equations do not balance dimensionally. If you must use mixed units, clearly state the units for each quantity in an equation. To demonstrate the effectiveness and feasibility of the proposed HWDO algorithm, six numerical examples from the literature are selected to conduct our simulation (as there are four groups of given parameters in the third example, it can be regarded as four different ones). Simulation results are also compared to the results acquired through the GA algorithm in the literature [6] and the PSO algorithm given in literature.

Example 1:

$$\begin{cases} \min\limits_{x} F(x,y) = 2x_1 + 2x_2 - 3y_1 - 3y_2 - 60 \\ s.t. 0 \le x_1 \le 50, 0 \le x_2 \le 50 \\ x_1 + x_2 + y_1 - 2y_2 \le 40 \\ \min\limits_{y} f(x,y) = (y_1 - x_1 + 20)^2 + (y_2 - x_2 + 20)^2 \\ s.t.\ 2y_1 - x_1 + 10 \le 0, 2y_2 - x_2 + 10 \le 0 \\ -10 \le y_1 \le 20, -10 \le y_2 \le 20 \end{cases}$$

Example 2:

$$\begin{cases} \min\limits_{x} F(x,y) = -x_1^2 - 3x_2 - 4y_1 + y_2^2 \\ s.t.\ x_1^2 + 2x_2 \le 4,\ x_1 \ge 0,\ x_2 \ge 0 \\ \min\limits_{y} f(x,y) = 2x_1^2 + y_1^2 - 5y_2 \\ s.t.\ x_1^2 - 2x_1 + x_2^2 - 2y_1 + y_2 \ge -3,\ x_2 + 3y_1 - 4y_2 \ge 4 \\ y_1 \ge 0,\ y_2 \ge 0 \end{cases}$$

Example 3:

$$\begin{cases} \min\limits_{x} F(x,y) = k(x_1^2 + x_2^2) - 3y_1 - 4y_2 + \frac{1}{2}(y_1^2 + y_2^2) \\ \min\limits_{y} f(x,y) = \frac{1}{2}[y_1\ y_2]H\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - [x_1\ x_2]\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \\ s.t.\ -0.333y_1 + y_2 - 2 \le 0,\ y_1 - 0.333y_2 - 2 \le 0 \end{cases}$$

(a) $k = 0.1$, $H = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$;

(b) $k = 1$, $H = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$;

(c) $k = 0$, $H = \begin{bmatrix} 1 & 3 \\ 3 & 10 \end{bmatrix}$;

(d) $k = 0.1$, $H = \begin{bmatrix} 1 & 3 \\ 3 & 10 \end{bmatrix}$。

In our simulation for the HWDO algorithm, the number of iterations and the air particles are set as 100 and 200 respectively. Our simulation results are illustrated in Table 1 and Table 2. As the upper-level

programming generally holds the leading role in the bi-level programming, the performance of the solving algorithm for the BLPP relies largely on the quality of the solution for the upper-level programming. Seen from Table 1, for all the six numerical examples, the global optimal solutions could be obtained through the proposed HWDO method. Of all the six, only the second example and the third(c) example's goal function values for the upper-level programming are slightly inferior to the results optimized through the other two methods. However, the global optimal solution for the second example lacks accuracy (JIANG Min, 2013). As mentioned by ZHAO Zhigang, DU Maokang and FU Deqiang (2007), with more advantages, the WDO method is more competitive than the PSO algorithm. Due to gravity, the air particles in the WDO algorithm are more likely to keep inside the optimizing space, and their robustness and stability are further enhanced by the Coriolis force from the Earth's rotation during the process of WDO's optimization. Consequently, all things considered, the advantages owned by the WDO algorithm make our proposed HWDO method much more proficient in searching ability. Meanwhile, the procedure of the HWDO method, as well as the simulation results, also proves that the forms of the bi-level programming's goal functions are not necessarily to be changed, and no other specific conditions, such as being differentiable, Kuhn-Tucker conditions, etc., are required. Therefore, as can be seen, the HWDO method is an effective and efficient way to solve the BLPP.

Table 1 Comparison of the optimizing outcomes

Eg.	Optimal value F of the upper-level programming			Optimal value f of the lower-level programming		
	GA[6]	PSO [11]	HWDO	GA[6]	PSO [11]	HWDO
Eg.1	5	0	-9.9566	0	100	58.7701
Eg.2	-12.68	-12.6787	-12.0064	-1.0156	-1.01563	-1.3176
Eg.3(a)	-8.9172	-8.9172	-8.9527	-6.137	-6.157	-12.2985
Eg.3(b)	-7.5782	-7.578458	-9.1796	-0.5738	-0.57192	-0.4844
Eg.3(c)	-11.9985	-11.9985	-11.9553	-163.42	-178.07	-390.6435
Eg.3(d)	-3.6	-3.6	-3.8783	-2	-3.6	-3.4468

Table 2 Solutions obtained by the WDO

Examples	Solution by the WDO(x1,x2,y1,y2)
Example 1	(21.2627, 16.6892, 5.6313, 2.9888)
Example 2	(0.1350, 1.9887, 1.6800, 0.8353)
Example 3(a)	(4.0721, 2.4034, 2.7970, 2.3933)
Example 3(b)	(1.1982, 0.3839, 2.7184, 2.1573)
Example 3(c)	(93.5873, 64.9060, 2.9108, 2.9601)
Example 3(d)	(4.2996, 0.4331, 2.1399, 0.4267)

CONCLUSION

Aiming at resolving the BLPP, a hierarchical wind driven optimization algorithm is put forward in this research. The framework of the hierarchical algorithm is composed of two WDO algorithms, whose interactive iterations are invoked to figure out the optimal solution of the BLPP. Simulated calculation outcomes of the numerical examples demonstrate that the HWDO method is accurate, effective and efficient. The simulation results also clarify that our HWDO based algorithm can not only work out the model of bi-level programming effectively, but also owns universality and better robustness, which will meet the demand of real-life decision-making problems in management. Additionally, different parameters' influence on the solution of the bi-level programming in the WDO process should be taken into consideration in the algorithm. Furthermore, the performance of the algorithm needs to be further tested by practical problems of large scale.

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