EFFECTS OF NON-EXTENSIVE Q-DISTRIBUTED ELECTRONS ON DUST ION ACOUSTIC SHOCK STRUCTURES IN AN UNBOUNDED DUSTY PLASMA WITH TWO THERMAL STATES OF ELECTRONS

[1]G. MANDAL, [2]M. ASADUZZAMAN

[1]Department of Mathematics and Physical Sciences,, East West University, Aftabnagar, Dhaka-1212, Bangladesh
[2]Institute of Natural Science, United International University, United City, Madani Avenue, Badda, Dhaka-1212, Bangladesh
E-mail: [1]gdmandal@ewubd.edu, [2]asad@ins.uiu.ac.bd

Abstract - Properties of dust ion-acoustic (DIA) shock waves in an unmagnetized dusty plasma consisting of inertial ions, nonextensive q-distributed two thermal states electrons, and negatively charged immobile dust are rigorously investigated using reductive perturbation technique. In this present investigation, Burgers equation is derived using ion continuity and momentum equations where two thermal states electrons provide restoring force. The Burgers equation contains a dissipative term arising from (i) the ion kinematic viscosity and (ii) the effects of two thermal states electrons.
These two terms, ion kinematic viscosity and the effects of two thermal states electrons, control the dynamics of DIA shock waves. Numerical analysis is represented with figures. It is observed noticeably that the non-extensive electron distribution can significantly change the potential profile and width of shock waves. Finally, the importances of DIA shock waves with two thermal states electrons which are present in different astro-plasma environments are briefly addressed.

Keywords - Dusty plasma, Dust ion-acoustic wave, Nonextensive q-distribution, Shock waves.

I. INTRODUCTION

Different types of nonlinear phenomena in dusty plasmas have become a very popular and important topic in research. It is experimentally found that the dust grains are present in comet tails, asteroid zones, planetary ring, interstellar medium, and the lower part of the Earths ionosphere [1]-[8]. This presence of dust grains (positively or negatively charged) in plasma can modify the collective behavior of plasma and generates new modes wave like dust ion acoustic (DIA) waves [9], dust acoustic (DA) waves [10]-[11], and dust lattice (DL) waves [12]. The study of dust-ion-acoustic waves has received more attention to the researchers, and it has been extensively studied both theoretically and experimentally. About two decades ago, Shukla and Silin [13] have theoretically shown that because of the equilibrium charge neutrality condition in dusty plasma, there is a strong inequality between ion number density and electron number density. Under this condition, a dusty plasma (with negatively charged immobile dust) supports the low-frequency dust-ion-acoustic wave. In this case, the ion mass provides the inertia and the restoring force comes from the electron thermal pressure. The existence of DIA waves has been verified by the laboratory experiments [14]-[15].
The theoretical and laboratory investigations on the DIA waves are done by a large numbers of authors [16]-[18], their investigations are appropriate only for a plasma situation where the plasma state is in equilibrium and the plasma species follow the well-known Maxwellian distribution. Equilibrium plasma state cannot provide any appreciable information, even it cannot explain the non-equilibrium stationary states [19]-[20], where nonextensivity plays a crucial role. Nowadays, nonextensive plasma [21] is a research topic of current interest due to its wide relevance in astrophysical and cosmological scenarios like stellar polytropes [19], hadronic matter and quark-gluon plasma [22], proton-neutron stars [23], dark-matter halos [24], etc. Nonextensive plasmas are investigated by many authors considering electron or ion or both to be nonextensively distributed [25]-[27]. Maxwellian distribution is inadequate to describe the long range interactions in unmagnetized collisionless plasma where the non-equilibrium stationary state exists. Space plasma observations clearly indicates the presence of ion and electron populations those are far away from their thermodynamic equilibrium [28]-[30]. Thus, incase of thermodynamic non-equilibrium situation nonextensive statistics or Tsallis statistics based on the derivation of Boltzmann-Gibbs-Shannon (BGS) entropic measure [31] is appropriate.
This non-extensive distribution first acknowledged by Renyi [32] and afterward it is proposed by Tsallis [31], where the entropic index q, characterized the degree of non-extensivity of the considered system.
However, in a plasma medium the dissipative properties can assist the formation of shock structures.
The Landau damping, kinematic viscosity among the plasma species, and the collision between ion-neutral, dust-neutral, etc. are the major causes for arising the dissipation which is responsible for the formation of shock structures in a plasma medium. The shock structures in dusty plasmas have been observed in a number of major research laboratories throughout the world. There are also plan to carry out practical experiments to study shock structure during the mission of the International Space Station. That is why, the propagation of DIA shock waves in dusty plasmas has a great significance to study. The DIA

shock waves in a dusty plasma have been observed by Nakamura [33]-[34] where he considered single temperature electrons. Most of the theoretical works on the DIA shock waves [33]-[35] are based on single-temperature electrons. However, the consideration of multiple-temperature electrons are also more realistic as such plasma environment exists both in space [36]-[37] and in laboratory [38]-[39].

In this paper, we have given more attention to analyze the different shock wave properties where electrons having two thermal states and also follow nonextensive q-distribution. This paper is organized as follows. The

basic equations are presented in Section II. The Burgers equation is derived in Section III. The basic features of

the shock waves are analyzed in Section IV. Finally, a brief discussion is presented in Section V.

II. GOVERNING EQUATIONS

We consider nonlinear propagation of the DIA waves in an unmagnetized dusty plasma system consisting of negatively charged stationary dust, inertial ions, and two thermal states of electrons following q-nonextensive distribution. At equilibrium we have $n_{io} = n_{e10} + n_{e20} + Z_d n_{d0}$, where n_{i0} is the equilibrium ion number density, n_{e10} (n_{e20}) is the number density of unperturbed higher (lower) temperature electron, Z_d is the number of electrons residing onto the dust grain surface. The nonlinear dynamics of the DIA waves, whose phase speed is much smaller (larger) than electron (ion) thermal speed, is governed by the following equations:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0, \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{\partial \psi}{\partial x} + \eta \frac{\partial^2 u_i}{\partial x^2}, \tag{2}$$

$$\frac{\partial^2 \psi}{\partial x^2} = 1 + G\psi + H\psi^2 - n_i. \tag{3}$$

where, $G = \frac{1}{2}(q+1)(\mu_1 + \sigma\mu_2)$, $H = \frac{1}{8}(q+1)(3-q)(\mu_1 + \sigma^2\mu_2)$, n_i is the ion number density normalized by its equilibrium value n_{i0}, u_i is the ion fluid speed normalized by $C_i = (k_B T_{e1}/m_i)^{1/2}$, η is the co-efficient of viscosity normalized by $m_i n_{i0} \omega_{pi} \lambda_{Dm}^2$, ψ is the electrostatic wave potential normalized by $k_B T_{e1}/e$, $\sigma = T_{e1}/T_{e2}$, $\mu_1 = n_{e10}/n_{i0}$, $\mu_2 = n_{e20}/n_{i0}$, and $\mu = Z_d n_{d0}/n_{i0} = 1 - \mu_1 - \mu_2$. It is noted that T_{e1} (T_{e2}) is the higher (lower) electron temperature, k_B is the Boltzmann constant, and e is the magnitude of the electron charge. The time variable t is normalized by $\omega_{pi}^{-1} = (m_i/4\pi n_{i0} e^2)^{1/2}$, and the space variable x is normalized by $\lambda_{Dm} = (k_B T_{e1}/4\pi n_{i0} e^2)^{1/2}$.

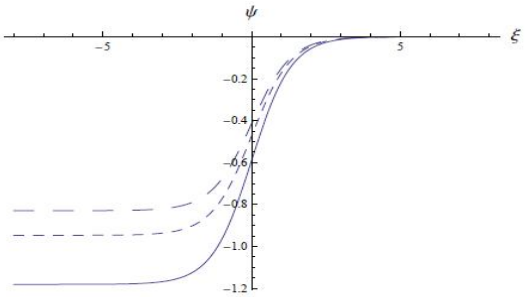


FIG. 1: Variation of potential ψ of shock waves with ξ for $\mu_1 = 0.2$, $\mu_2 = 0.6$, $\sigma = 1.2$, $\eta = 0.4$, $q = 0.2$ (solid curve), $q = 0.4$ (dotted curve), $q = 0.6$ (long-dashed curve)

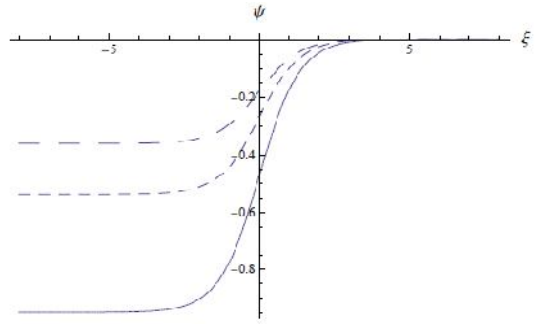


FIG. 2: Variation of potential ψ of shock wave with ξ for different values of $\sigma = 1.2$ (solid curve), $\sigma = 1.4$ (dotted curve) and $\sigma = 1.6$ (long-dashed curve). where $\mu_1 = 0.2$, $\mu_2 - 0.6$, $\eta - 0.4$, $q - 0.4$

To take into account of nonextensivity of electrons, we use the following q-distribution

$$n_{e1} = n_{e10}[1 + (q-1)\frac{e\phi}{k_B T_{e1}}]^{\frac{1}{q-1}+\frac{1}{2}} \tag{4}$$

$$n_{e2} = n_{e20}[1 + (q-1)\frac{e\phi}{k_B T_{e2}}]^{\frac{1}{q-1}+\frac{1}{2}} \tag{5}$$

III. DERIVATION OF BURGERS EQUATION

We first derive the Burgers equation from our basic equations (1)-(5). We employ the reductive perturbation technique (RPT) and the stretched coordinates for the derivation of modified Burgers equation. Stretched coordinates:

$$\xi - \epsilon(x - v_p t), \tag{6}$$

$$\tau - \epsilon^2 t, \tag{7}$$

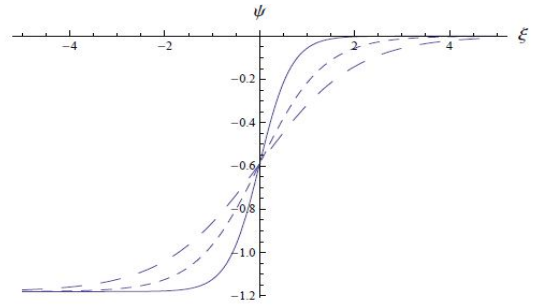


FIG. 3: Variation of potential ψ of shock wave with ξ for different values of $\eta = 0.2$ (solid curve), $\eta = 0.4$ (dotted curve), $\eta = 0.6$ (long-dashed curve) where $\mu_1 = 0.2$, $\mu_2 = 0.6$, $\sigma = 1.2$, $q = 0.4$

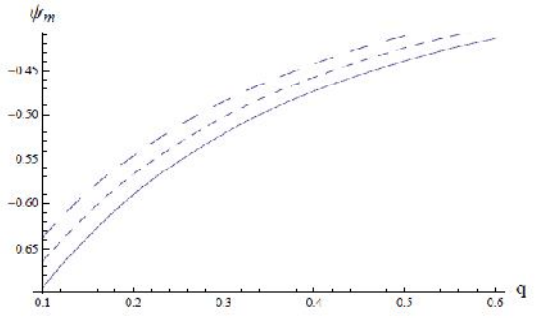


FIG. 4: Variation of the wave amplitude ψ_m with nonextensive parameter q for $\mu_1 = 0.2$, $\mu_2 = 0.6$, $\eta = 0.4$, $\sigma = 1.10$ (solid curve), $\sigma = 1.11$ (dotted curve) $\sigma = 1.12$ (long-dashed curve)

where ϵ is a small parameter ($0 < \epsilon < 1$) measuring the weakness of the dispersion, and V_p (normalized by C_i) is the phase speed (ω/k) of the perturbation mode. The variables n_i, u_i, and ψ can be expanded in power series of ϵ, viz.

$$n_i = 1 + \epsilon n_i^{(1)} + \epsilon^2 n_i^{(2)} + \epsilon^3 n_i^{(3)} + \cdots, \quad (8)$$

$$u_i = 0 + \epsilon u_i^{(1)} + \epsilon^2 u_i^{(2)} + \epsilon^3 u_i^{(3)} + \cdots, \quad (9)$$

$$\psi = 0 + \epsilon\psi^{(1)} + \epsilon^2\psi^{(2)} + \epsilon^3\psi^{(3)} + \cdots. \quad (10)$$

Now, expressing Eqs. (1)-(3) in terms of ξ and τ, and substituting Eqs. (6)-(8) into the resulting equations, one can easily develop different sets of equations in various powers of ϵ. To the lowest order in ϵ one obtains

$$u_i^{(1)} = \frac{1}{V_p}\psi^{(1)}, \quad (11)$$

$$n_i^{(1)} = \frac{1}{V_p^2}\psi^{(1)}. \quad (12)$$

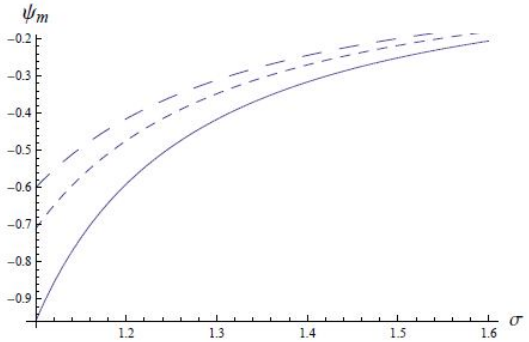


FIG. 5: Variation of the wave amplitude ψ_m with σ for $\mu_1 = 0.2$, $\mu_2 = 0.6$, $\eta = 0.4$, $q = 0.2$ (solid curve), $q = 0.4$ (dotted curve) $q = 0.6$ (long-dashed curve)

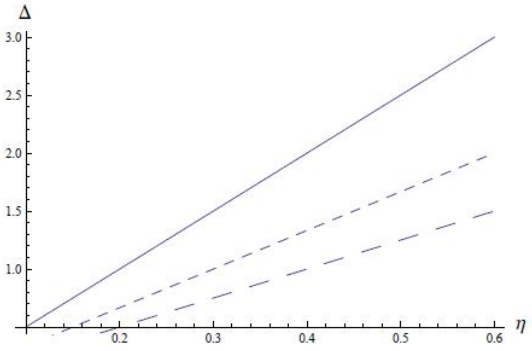


FIG. 6: Variation of the width δ of the wave with η for $\mu_1 = 0.2$, $\mu_2 = 0.6$, $\sigma = 1.2$, $q = 0.4$, $U_0 = 0.2$ (solid curve), $U_0 = 0.3$ (dotted curve), $U_0 = 0.4$ (long-dashed curve)

$$V_p = \frac{1}{\sqrt{\frac{1}{2}(q+1)(\mu_1 + \sigma\mu_2)}}, \quad (13)$$

Equation (13) represents the phase speed of the DIA waves propagating in a dusty plasma under consideration where population of two thermal electrons exist. To the next higher order in ϵ, we obtain a set of equations, which, after using Eqs. (11)-(13), can be simplified as

$$\frac{\partial n_i^{(1)}}{\partial\tau} - V_p\frac{\partial n_i^{(2)}}{\partial\xi} + \frac{\partial u_i^{(2)}}{\partial\xi} + \frac{\partial}{\partial\xi}\left[n_i^{(1)}u_i^{(1)}\right] = 0, \quad (14)$$

$$\frac{\partial u_i^{(1)}}{\partial\tau} - V_p\frac{\partial u_i^{(2)}}{\partial\xi} + u_i^{(1)}\frac{\partial u_i^{(1)}}{\partial\xi} + \frac{\partial\psi^{(2)}}{\partial\xi} - \eta\frac{\partial^2 u_i^{(1)}}{\partial\xi^2} = 0, \quad (15)$$

$$G\frac{\partial\psi^{(2)}}{\partial\xi} + 2H\psi^{(1)}\frac{\partial\psi^{(1)}}{\partial\xi} - \frac{\partial n_i^{(2)}}{\partial\xi} = 0. \quad (16)$$

Now, combining Eqs. (14)-(16), we obtain a new equation of the form:

$$\frac{\partial\psi^{(1)}}{\partial\tau} + A\psi^{(1)}\frac{\partial\psi^{(1)}}{\partial\xi} = C\frac{\partial^2\psi^{(1)}}{\partial\xi^2}, \quad (17)$$

where A is the nonliear coefficient and C is the dissipative coefficient. The expressions for the coefficients A and C are:

$$A = V_p^3\left[\frac{3}{2}\frac{1}{V_p^4} - H\right], \quad (18)$$

$$C = \frac{\eta}{2}. \quad (19)$$

Equation (17) is well known Burgers equation, derived for the DIA shock waves in dusty plasma. For a frame moving with a speed U_0, the stationary shock wave solution of eqs. (17) is

$$\psi^{(1)}(\equiv\psi) = \psi_m\left[1 - \tanh\left(\frac{\xi}{\Delta}\right)\right], \quad (20)$$

where $\Delta = 2C/U_0$ is the width, and $\psi_m = U_0/A$ is the amplitude of the shock wave.

IV. NUMERICAL RESULTS AND GRAPHICAL PRESENTATION

We have numerically analyzed the wave amplitude, the width of the wave and the potential (ψ) profile of the shock wave considering the following set of parameters: $\mu_1 = n_{e10}/n_{i0} = 0.2$, $\mu_2 = n_{e20}/n_{i0} = 0.6$, $\sigma = T_{e1}/T_{e2} = 1.2 \quad 1.6$, $\eta = 0.2 \quad 0.4$, $q = 0.1 \quad 0.6$.

For the above set of parameters we found that the shock wave potential is always negative. Negative potential indicates the shock waves are rarefactive. We consider the two thermal states electrons and it is found that this variation of temperature can have significantly effects on shock wave potential. Effects of kinematic viscosity are also investigated in the present work. It is very interesting that there is a common potential (-0.6) although figures are plotted by considering different values of η.

The results are displayed in figures $1 - 6$.

Figure1 shows the variation of the solution ψ of the burgers's equation (20) with ξ for different values of nonextensive parameter q. We observe from the figue1 that as we increase the value of q the potential of the shock wave decreases.

Figure2 shows the variation of the solution ψ of the burgers's equation (20) with ξ for different values of σ as we increase the value of σ the potential of the shock wave decreases. If we increase the temperature of electron with first thermal state then it can change the potential of shock waves.

Figure3 shows the variation of the solution ψ of the burgers's equation (20) with ξ for different values of η. The kinematic viscosity has significant effect on potential. Fig3 indicates that on the positive side of ξ potential increases with the increase of η but potential decreases with the increase of η on the negative side of ξ.

Figure4 shows the variation of amplitude (ψ_m) of the shock wave against the nonextensive parameter q for different values of σ. Fig4 shows that if we increase the value of q then amplitude (ψ_m) also increases. At the same time there is a very slightly variation found for different values of σ.

Figure5 shows the variation of amplitude (ψ_m) of the shock wave against the σ for different values of q. If we increase σ then amplitude of the shock waves increases. Even if we increase the value of q then also there is no significant effect on amplitude ψ_m.

Figure6 shows the variation of width (δ) of the shock wave against the η for different values of U_0. Kinematic viscosity η has important effect on with of the shock waves. When we increase the η, width of shock waves increases. On the other hand, if we increase the frame velocity U_0 then width decreases.

V. DISCUSSIONS

1. Rarefactive shock waves are formed in dusty plasma for our considered parameters set: $\mu_1 = n_{e10}/n_{i0} = 0.2$, $\mu_2 = n_{e20}/n_{i0} = 0.6$, $\sigma = T_{e1}/T_{e2} = 1.2 - 1.6$, $\eta = 0.2 - 0.4$, and $q = 0.1 - 0.6$..
2. It is observed that the nonextensive parameter q has significant effects on shock waves.
3. The variation σ which is the ratio between two thermal states of electrons is also very important on potential of shock waves.
4. The amplitude of shock waves increases with the increase of nonextensive parameter q and σ
5. It has been observed that the width of shock waves increases with the increase of η.

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