OSCILLATORY FLOW OF AIR OVER MAGNETIC FLUID COATED ON OSCILLATING FLAT PLATE

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Abstract- This study deals with an oscillatory flow of air over a uniform magnetic fluid coating on a flat plate oscillating in its own plane. The steady state solution of the oscillatory flow problem in air and magnetic fluid satisfying the boundary and the interface conditions are obtained using a complex shape function, from which the distributions of velocity and shear stress are determined. Special attention is paid to the behaviors of the velocity and the shear stress profiles and the time evolution of interface velocity, interface shear stress, and wall shear stress for varying oscillation frequency. The results for a particular set of density and viscosity ratios of 0.001 and 0.004 show that the flow decays exponentially in space with a rate proportional to the frequency parameter. The flow variables at the interface and the wall exhibit different behaviors with respect to the frequency parameter.

Keywords- Air, Flat Plate, Frequency, Magnetic Fluid Coating, Steady State Oscillation.

I. INTRODUCTION

Like other functional fluids such as electrorheological fluid and magnetorheological fluid, magnetic fluid, sometimes called ferrofluid, is a deliberately synthesized fluid. It is a colloidal suspension of nanoscale magnetic particles dispersed in a non-magnetic base liquid such as water or organic solvent [1]-[2]. Clumping of magnetic particles can be avoided by coating each particle with a surfactant. Since magnetic fluid has properties like magnetic material its flow can be controlled by externally applying magnetic field. It has many application fields including sealing, magnetic bearings, and actuators. It is also used in the biomedical field to trace blood flow or to control drug delivery [3]-[5]. When the magnetic fluid is coated on solid surface in the flow fields of normal fluid, the whole flow and heat transfer characteristics can be changed by externally applying a magnetic field, and a reduction of drag or enhancement of heat transfer is possible. By coating a magnetic fluid on a circular cylinder in a uniform flow, Krakov and Kamiyama [6] observed a remarkable drag reduction. In this flow the drag reduction resulted from the reduced wake area which was possible since the magnetic fluid coating retarded flow separation. The drag reduction ratio was shown to depend on the viscosity ratio and the Reynolds number. Ko et al. [7] numerically studied a stagnation point flow over a plane wall having a magnetic fluid coating of uniform thickness and showed the magnetic fluid coating reduces the wall shear stress. Magnetic fluid coating is also applicable to control the convection heat transfer. In an experimental investigation of the heat transfer and the flow structure in a flat channel coated with magnetic fluid of sinusoidal shape Bashtovoi et al. [8] observed that the magnetic fluid coating reduces drag as well as enhances heat transfer. The possibility of heat tranfer enhancement by using a magnetic fluid coating was numerically investigated for a channel with a fin and the changed flow pattern owing to the coating was able to increase the convection heat transfer [9].

Because of the nonlinearity exactly solving the Navier-Stokes equation is difficult and only a limited number of exact solutions are known for the flows in a simple geometry. Most of the exact solutions of theNavier-Stokes equation can be classified into two groups of similarity solutions and unidirectional flows. Out of the latter group of flows, a viscous flow resulting from the in-plane oscillation of a plane wall is called the Stokes' second problem [10]-[11]. As the flow acts a guide to the understanding of many complicated flows and occurs in many application problems it is not only interesting but also important. By using the Laplace transform method Erdogan [12] and afterwards Fetecau et al. [13] solved the Stokes' second problem and found a transient solution as well as steady-state solution starting from a rest state. The similar unsteady problems were solved for the non-Newtonian fluids in [14]-[15]. Recently Ko and Kim [16] investigated the effect of frequency on the steady state oscillatory flow of liquid over a magnetic fluid which is coated on an oscillating flat plate with uniform thickness.

In this study a Stokes' second problem is considered for air over a magnetic fluid which is coated with uniform thickness on a flat plate oscillating in its own plane. This study is complementary to [16] in that the gas-liquid combination is considered. In terms of several parameters such as density ratio, viscosity ratio, and the frequency parameter, the steady state oscillatory flow is obtained in a closed form from which the velocity and shear stress distributions are readily determined. Special attention is paid to the effect of the frequency parameter on the flow characteristics. The features of flow including the space and time modulations of velocity and shear

stress and the time evolution of velocity and shear stress at the wall and interface are presented for a particular set of density and viscosity ratios.

II. FORMULATIONAND ANALYSIS

The schematic illustration of the flow problem is shown in Fig. 1. Air rests over a magnetic fluid coated on a flat plate with uniform thickness of *b*. The height of the magnetic fluid coating are kept constant with respect to both space and time because strong enough external magnetic field is applied to hold it. A unidirectional flow in the direction parallel to the plate, namely in the *x* direction, is generated by a harmonic oscillation of the flat plate in its own plane. The resulting flows in the magnetic fluid and air can be assumed incompressible and laminar. As air is non-magnetic the magnetic field only affects the magnetic fluid. In this flow the only effect of the magnetic force acting on the magnetic fluid is the modifying of the pressure field. As in the Stokes' second problem, the velocity field can be decoupled from the pressure field since neither modified pressure (sum of the mechanical pressure and the magnetic pressure) of the magnetic fluid nor pressure of air varies in the *x* direction.

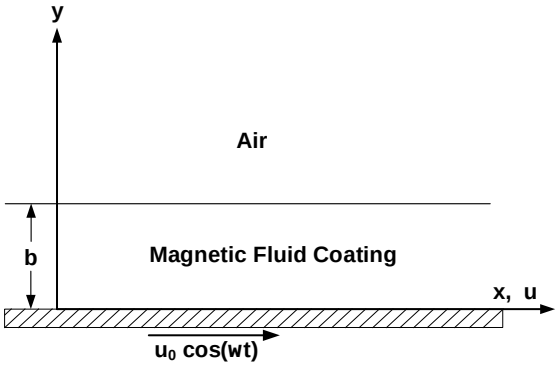


Fig. 1. Schematic illustration of the flow problem.

It is well known that the unidirectional flows automatically satisfy the continuity equation. If we denote the magnetic fluid and air by the subscripts 1 and 2, respectively, then the momentum equation governing the flow, and the boundary conditions and the interface conditions are written as follows:

$$\rho_1 \frac{\partial u}{\partial t} = \mu_1 \frac{\partial^2 u}{\partial y^2} \quad (0 < y < b)$$

(1a)

$$\rho_2 \frac{\partial u}{\partial t} = \mu_2 \frac{\partial^2 u}{\partial y^2} \quad (b < y)$$

(1b)

$$u(0,t) = u_0 \cos \omega t$$

(2a)

$$u(\infty,t) = 0$$

(2b)

$$u(b-0,t) = u(b+0,t)$$

(3a)

$$\mu_1 \frac{\partial u(b-0,t)}{\partial y} = \mu_2 \frac{\partial u(b+0,t)}{\partial y}$$

(3b)

Here, $u(y, t)$ is the velocity of the fluids, and y and t denote the coordinate normal to the flat plate and time, respectively. In addition □ and □ mean the density and the viscosity of the fluids, and u_0 and □ are the amplitude and the frequency of harmonic oscillation of the wall velocity. The conditions in (3a) and (3b) are imposing the continuity of velocity and the continuity of shear stress at the interface, respectively.

For the convenience of calculation and presentation, the system of equations from (1a) to (3b) are transformed to a non-dimensional form by introducing the following dimensionless variables.

$$U = u/u_0 \quad (4)$$

$$T = \omega t$$

(5)

$$Y = y/b$$

(6)

The resulting non-dimensional system of governing equations is as follows:

$$\mathrm{Re}_1 \frac{\partial U}{\partial T} = \frac{\partial^2 U}{\partial Y^2} \quad (0 < Y < 1)$$

(7a)

$$\mathrm{Re}_2 \frac{\partial U}{\partial T} = \frac{\partial^2 U}{\partial Y^2} \quad (1 < Y)$$

(7b)

$$U(0,T) = \cos T$$

(8a)

$$U(\infty,T) = 0$$

(8b)

$$U(1-0,T) = U(1+0,T)$$

(9a)

$$\frac{\partial U(1-0,T)}{\partial Y} = \mu_r \frac{\partial U(1+0,T)}{\partial Y}$$

(9b)

Here $\mathrm{Re}_1 = \rho_1 \omega b^2/\mu_1$ and $\mathrm{Re}_2 = \rho_2 \omega b^2/\mu_2$ are the Reynolds numbers for the flows in the magnetic field and air and $\mu_r = \mu_2/\mu_1$ is the ratio of the viscosity of air to that of the magnetic fluid.

Considering the linearity of the system and the boundary condition, (8a), the steady state solution to the system of equations from (7a) to (9b) can be sought in the following form [14], [16].

$$U(Y,T) = \mathrm{Re}\left[e^{iT} F(Y)\right]$$

(10)

Here Re means the real part and *i* is the imaginary unit. Substituting (10) into the system of equations for *U* and performing a little tedious calculationthe final solution for the velocity filed are obtained as follows.

$$U = \mathrm{Re}\left[A_1 e^{-\lambda_1 Y + i(T-\lambda_1 Y)} + (1-A_1) e^{\lambda_1 Y + i(T+\lambda_1 Y)}\right] \quad (0<Y<1) \qquad (11a)$$

$$U = \mathrm{Re}\left[A_2 e^{-\lambda_2 Y + i(T-\lambda_2 Y)}\right] \quad (1<Y) \qquad (11b)$$

$$A_1 = \left[1-\alpha e^{-2\lambda_1(1+i)}\right]^{-1} \qquad (12)$$

$$A_2 = (1-\alpha) e^{(\lambda_2-\lambda_1)(1+i)} A_1 \qquad (13)$$

$$\alpha = \left(\sqrt{\rho_r \mu_r} - 1\right) \Big/ \left(\sqrt{\rho_r \mu_r} + 1\right) \qquad (14)$$

$$\lambda_1 = \sqrt{\mathrm{Re}_1 / 2} \qquad (15)$$

$$\lambda_2 = \sqrt{\mathrm{Re}_2 / 2} = \lambda_1 \sqrt{\rho_r / \mu_r} \qquad (16)$$

$$\rho_r = \rho_2 / \rho_1 \qquad (17)$$

The remaining variables of flow such as shear stress distribution and interface velocity can be easily calculated from this solution. More specifically the non-dimensional shear stress defined by

$$S = \frac{\tau}{\mu_2 u_0 / b} = \frac{\mu_j}{\mu_2} \frac{\partial U}{\partial Y} \quad (j = 1, 2) \qquad (18)$$

can be obtained by differentiating the velocity given in (11a) and (11b) as follows.

$$S = \frac{\sqrt{2}\lambda_1}{\mu_r} \mathrm{Re}\left[-A_1 e^{-\lambda_1 Y + i(T-\lambda_1 Y + \frac{\pi}{4})} + (1-A_1) e^{\lambda_1 Y + i(T+\lambda_1 Y + \frac{\pi}{4})}\right] \quad (0<Y<1) \qquad (19a)$$

$$S = -\sqrt{2}\lambda_2 \,\mathrm{Re}\left[A_2 e^{-\lambda_2 Y + i(T-\lambda_2 Y + \frac{\pi}{4})}\right] \quad (1<Y) \qquad (19b)$$

The velocity and the shear stress of the magnetic fluid and air are same at their interface. Hence they are calculated more easily from (11b) and (19b), respectively, and equal to

$$U_i = \mathrm{Re}\left[A_2 e^{-\lambda_2 + i(T-\lambda_2)}\right] \qquad (20)$$

$$S_i = -\sqrt{2}\lambda_2 \,\mathrm{Re}\left[A_2 e^{-\lambda_2 + i(T-\lambda_2 + \frac{\pi}{4})}\right] \qquad (21)$$

Finally, the shear stress at the wall is expressed as

$$S_w = \frac{\sqrt{2}\lambda_1}{\mu_r} \mathrm{Re}\left[(1-2A_1) e^{i(T+\frac{\pi}{4})}\right] \qquad (22)$$

III. RESULTS AND DISCUSSION

This study conducts an investigation of an oscillatory flow of air over a magnetic fluid which is coated on a flat plate with uniform thickness. The steady in-plane oscillation of the plate generates the motion of the magnetic fluid, which causes the air flow. This study is complementary to the study of the oscillatory liquid flow over a magnetic fluid coating on an oscillating flat plate [16]. In [16] the velocity and shear stress fields for the conventional solution of the Stokes' second problem are recovered both in the fluid and in the magnetic fluid from a special case of the density and viscosity ratios. In the last section a closed form solution corresponding to the steady state oscillating flow for arbitrary combination of fluids is obtained by using a complex shape function. Three independent parameters which determines the oscillatory air flow over magnetic fluid coating are density ratio ρ_r, viscosity ratio μ_r, and frequency parameter λ_1. It is noted that the non-dimensional flow is utterly independent of the oscillation amplitude u_0 and the Reynolds number Re_1 based on the characteristic velocity of ωb is equal to $2\lambda_1^2$.

Both density and viscosity of gases are a couple of hundred times smaller than liquids in general. In this study a particular set of density and viscosity ratios is considered for the investigation of the effect of the frequency parameter λ_1 on the oscillatory air flow : ρ_r = 0.001, μ_r = 0.004. These values correspond to the combination of water based magnetic fluid (ρ= 1220 kg/m^3, μ= 4.88x10^{-3} N·s/m^2) which was used in the experiment of [6] and air (ρ= 1.23 kg/m^3, μ= 1.80x10^{-5} N·s/m^2). The frequency parameter λ_1 is considered for a range of 0.25 to 4 and the case of λ_1 = 1 will be referred to a base case. When the thickness of the water based magnetic fluid coating is 1 mm, the frequency of oscillation corresponding to the base case is 8 rad/s = 76 rpm. It is to be noted that λ_1 is proportional to the square root of ω.

The effects of the oscillation frequency on the velocity field are shown in Fig. 2 for three representative values of λ_1: (a)λ_1= 0.5, (b) λ_1= 1, and (c)λ_1= 2. If the oscillation frequency of the base case (λ_1= 1) is equal to 76 rpm, then those of (a) and (c) are equal to 19 rpm and 304 rpm, respectively. The velocity profiles are shown with a time increment of π/6 over a half period. First of all three profiles show the same pattern at the wall, Y = 0, and the satisfaction of the no slip boundary condition is well

assured. Since air and magnetic fluid have very different viscosities, discontinuity of velocity gradient is easily

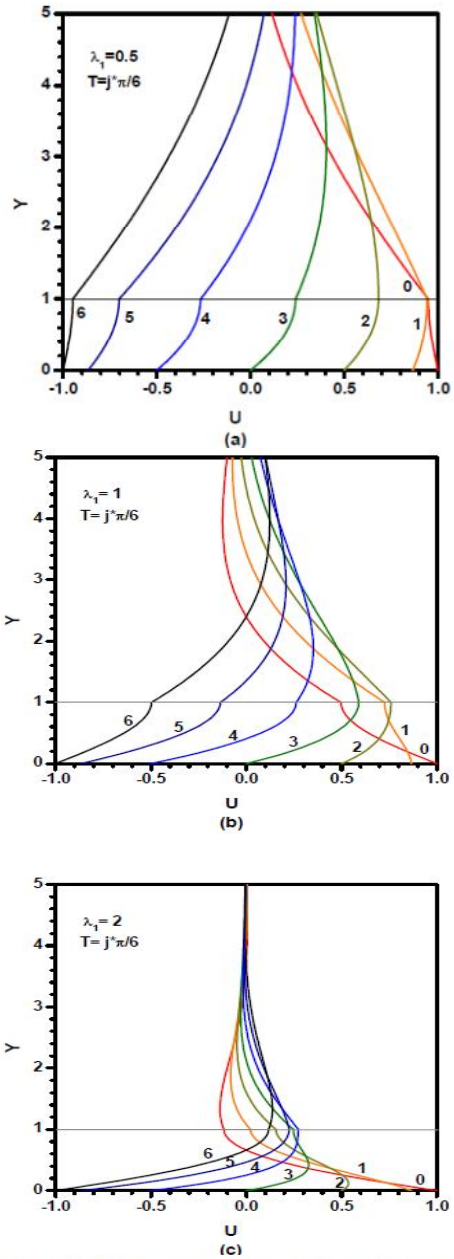


Fig. 2. Velocity profiles shown for π/6 increments over a half period for the case of (a) $\lambda_1 = 0.5$, (b) $\lambda_1 = 1$, and (c) $\lambda_1 = 2$.

observed at the interface regardless of the oscillation frequency at every instant. More specifically the velocity gradient of the magnetic fluid nearly zero at the interface and finite values of the velocity gradient of air begin to set up there. At any instant of time the change of velocity with respect to space occurs in two ways: the amplitude modulation and the phase modulation. They are revealed through the terms $\exp(\pm\lambda_1 Y)$ and $\exp(-\lambda_2 Y)$, and the terms $\cos(T\pm\lambda_1 Y +$ const.) and $\cos(T-\lambda_2 Y +$ const.), respectively. From (16), note that $\lambda_2 = 0.5\lambda_1$. In the magnetic fluid the modulation is very complicated since several factors coexist. Anyway, the magnitude of modulation becomes bigger as λ_1 increases. For λ_1 =2, a reversal of flow is observed at the instants corresponding to j = 0, 6 in Fig. 2 (c). For higher enough values of λ_1 multiple reversal of flow is possible. As the oscillation frequency gets lower, the flow in the lower region becomes more and more similar to the plane Couette flow. The modulation in air is simpler; the amplitude decreases according to a factor $\exp(-0.5\lambda_1 Y)$ and the phase decreases by an amount of $0.5\lambda_1 Y$. Therefore the oscillatory flow of air rapidly diminishes for larger values of λ_1. Every increase of $\ln 4/\lambda_1$ in Y halves the velocity amplitude. If the thickness of the oscillating layer is defined where $U = 0.01$, it is larger than one for all cases considered in Fig. 2; in other words, the interface velocity is faster than 1% of the wall velocity. The thickness of the oscillating layer decreases with the increase of oscillation frequency as in the flow of the conventional Stokes' second problem.

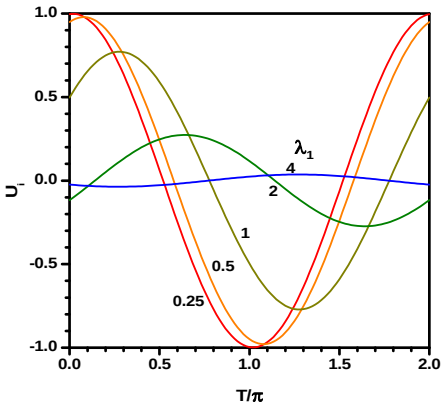


Fig. 3. Time evolution of the interface velocity over one period for various values of λ_1.

Fig. 3 depicts the time evolution of the interface velocity U_i over a full period of oscillation for the varying values of λ_1 between 0.25 and 4. Recall that $U = \cos T$ at the wall ($Y = 0$), and the interface corresponds to the line, $Y = 1$. The instantaneous velocity at the interface shows a simple harmonic oscillation with an amplitude less than 1 and a positive phase. The amplitude and phase modulations are dependent on the oscillation frequency. For low values of λ_1 (0.25 and 0.5) both modulations are small: values of the amplitude and the phase are nearly 1 and 0, respectively. As $\lambda_1$increases, the amplitude diminishes and the phase increases rather quickly. For λ_1= 2the interface velocity has an amplitude of about 0.27 and a phase of about 0.63π. For the highest value of λ_1= 4 of the considered the amplitude drops below 4%. It is expected that the oscillation layer is confined in the lower layer for the much higher values of λ_1. If the frequency is not extremely high, the oscillation of the flat plate propagates to air by the viscous effect of the magnetic fluid.

Fig. 4 illustrates the effects of the oscillation frequency on the shear stress distribution for three representative values of values of λ_1: (a)λ_1= 0.5, (b) λ_1= 1, and (c)λ_1= 2. Note that the frequency becomes four times when λ_1 doubles. Comparing with the base case of (b), case (a) represents the cases where the flat plate oscillates with a low frequency, and case (c)

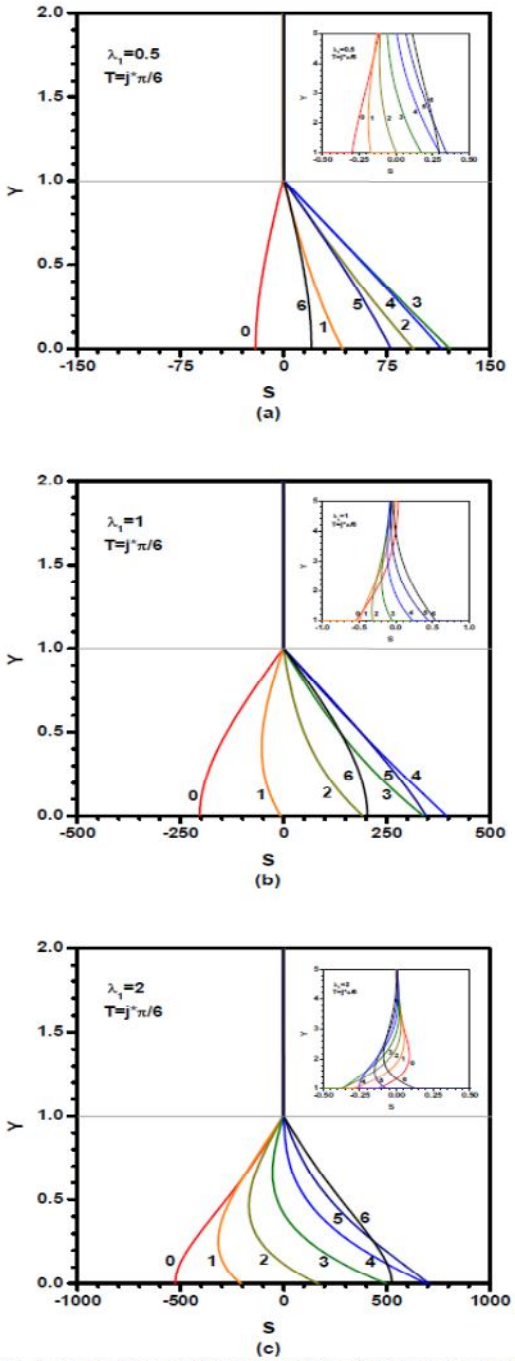


Fig. 4. Shear stress profiles shown for π/6 increments over a half period for the case of (a) $\lambda_1 = 0.5$, (b) $\lambda_1 = 1$, and (c) $\lambda_1 = 2$.

represents the cases of oscillation with a high frequency. The shear stress profiles are shown with a time increment of $\pi/6$ over a half period. As air is 250 times less viscous than the magnetic fluid and the times less viscous than the magnetic fluid and the shear stress is proportional to the viscosity, its values in the upper layer are too small to distinguish in a graph. Because of very small shear stress, air can be approximated stress free. Therefore magnified auxiliary plots are shown in the upper right section for a range of Y between 1 and 5 with a scale different from the main plot. The oscillation amplitudes of shear stress for the cases (a), (b), and (c) diminishes rather rapidly from hundreds (100, 400, 700) at the wall to less than one (0.35, 0.55, 0.38) at the interface. Like the velocity field the shear stress distribution exhibits a pattern of damped harmonic oscillation. The amplitude modulation depends on the frequency in a manner which is more complicated than the velocity field. The reason is because the shear stress is proportional to the product of viscosity and velocity gradient. But, its spatial dissipation rate in air is higher for higher frequency (higher value of λ_1). The amplitude in air is proportional to $\exp(-0.5\lambda_1 Y)$ with a proportionality constant depending on λ_1. The phase modulation of the shear stress, however, is similar to that of velocity since it is manifested as a linear function of Y. For λ_1 =2, a sign change of shear stress is observed at the instants corresponding to j = 2, 3 in Fig. 4 (c).

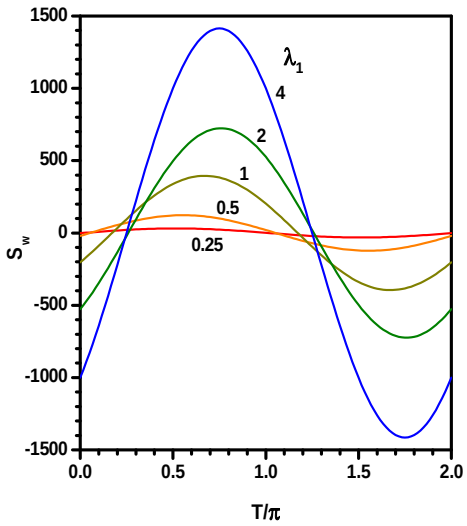


Fig. 5. Time evolution of the wall shear stress over one period for various values of λ_1.

The time evolution of the wall shear stress S_w over a full period of oscillation is shown in Fig. 5 for the varying values of λ_1 between 0.25 and 4. Wall shear stresses also exhibit harmonically oscillating behavior. The amplitude of oscillation is monotonically increasing with the increase of λ_1, while the dependence of phase on λ_1 is relatively weak. For λ_1 = 4, the phase shift of wall shear stress with respect to wall velocity is 0.75π which is the value corresponding to the conventional Stokes' second problem.

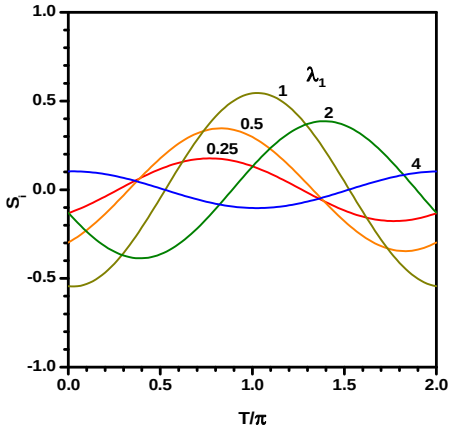


Fig. 6. Time evolution of the interface shear stress over one period for various values of λ_1.

Fig. 6 depicts the time evolution of the interface shear stress S_i over a full period of oscillation for the varying values of λ_1 between 0.25 and 4. Like the wall shear stress and interface velocity S_i also has a harmonically oscillating pattern with respect to time. As is observed in Fig. 4, the amplitude of oscillation is not monotonic but exhibits an increasing and then decreasing behavior with respect to the frequency. A rough estimation implies that it is nearly proportional to $\lambda_1 \exp(-0.5\lambda_1)$, and reaches a peak at $\lambda_1 \approx 1.1$. In addition its magnitude is much smaller than that of the wall shear stress, which is the result of a viscous dissipation in the magnetic fluid. The phase of oscillation increases almost linearly with respect to λ_1.

CONCLUSIONS

An analytical investigation of the oscillatory air flow over a magnetic fluid uniformly coated on a flat plate is carried out, where the flow is generated by the harmonic oscillation of the plate. The steady state distributions of velocity and shear stress are obtained by using a complex shape function. The features of the oscillatory flow are investigated for a varying frequency parameter and a particular set of density and viscosity ratios of 0.001 and 0.004, respectively. The oscillating flow shows a harmonic oscillation with an exponentially decreasing amplitude and a linearly varying phase shift in space. The interface velocity and the wall shear stress monotonically decreases and increases with the frequency parameter, respectively. The interface shear stress has a peak value with respect to the frequency parameter.

A more extensive study for the varying viscosity ratio is helpful for the full understanding of the characteristics of oscillating air flow over a magnetic fluid coating. In spite of its mathematical difficulty, the solution to the fully transient version of that problem is challenging.

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