ADAPTIVE CONTROL FOR A PIEZOELECTRIC-ACTUATING TABLE BASED ON LUGRE FRICTION MODEL WITH FUNCTIONAL APPROXIMATION COMPENSATION

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Abstract - A long range friction actuating mechanism was designed by using piezoelectric material to generate high frequency oscillation for actuating a finger tip which contacted with a slide to induce the back and forth motion. The LuGre friction model is chosen to simulate the dynamics of this friction actuating mechanism. However, this piezoelectric actuating system has obvious nonlinear and time-varying dead-zone offset control voltage due to the static friction and preload. It is difficulty to establish an accurate dynamic model for model-based precision control design. Hence, the functional approximation (FA) scheme is employed to compensate the system modeling error. The Laypunov-like design strategy is adopted to derive the adaptive laws and the system stability criterion. The dynamic experimental results of the proposed controller are compared with that of a model-based PID controller.

Keywords - Piezoelectric actuator, LuGre friction model, FA based adaptive control.

I. INTRODUCTION

Due to the precision manufacturing requirements of semi-conductor related industry, the precise position control technology and positioning mechanism are important developing targets. Hence, the piezoelectric actuating element is the key component of micro/nano level positioning system. Piezoelectric material has the advantages of small size, quick response speed, high transformation efficiency between mechanical and electric properties, and accurate resolution. It has been widely selected as an actuator of micro positioning table. However, it has the disadvantages of small travel, serious hysteretic behavior and small tension or torsion capability. In order to improve the defects of small travel and low tension capability, a friction actuating mechanism was designed by choosing piezoelectric material to generate high frequency oscillation for actuating a finger tip which contacted with a slide to induce the back and forth motion. This piezoelectric actuating X-Y table is manufactured by Nanomotion Ltd. Company with 100 mm travel range. Under a designed 39.6 KHz sinusoidal input voltage wave form, the combination of the PZT element longitudinal and bending deformations will induce the finger tip back and forth motion. The friction force between the front head of the finger tip and the slide contact surface is used to drive the linear motion of the slide. However, the static friction due to preload and motion contact between the finger tip and slide has caused an obvious starting offset control voltage. It is called as dead-zone offset control voltage in this study. It is a time-varying and temperature dependent value. If this friction phenomenon has not appropriately modeled and considered in the controller design, it will cause the oscillation and large tracking error in transient response and steady state error. The literature reviews of friction problem study and friction models investigation were proposed in [1]. The friction behaviors can be divided into static friction, coulomb friction, stick friction stricbeck effect and hysteretic phenomenon. Dahl [2] proposed a Dahl model to describe the spring-like starting friction behavior. However, it can not describe the stricbeck friction effect. Canudas et al. [3] extended Dahl model to include stricbeck effect, hysteretic and varying break-away force and proposed a LuGre friction model. It is widely employed in the friction simulation and model-based control compensation. The control strategies of friction compensation can be classified into model-based and model-free two categories. Liu [4] proposed a decomposition-based friction compensation method and adaptive/robust control schemes to compensate the model uncertainties. Zhang [5] employed LuGre model to design an adaptive control for nonlinear friction compensation. Choi [6] proposed a modified LuGre model by using a neural network Preisach model approach to handle the non-local memory hysteretic phenomenon. In addition, neural network and fuzzy control strategies were employed to design model-free friction compensators [7-11]. The friction force is considered as a time-varying unknown estimation component without model information. Lin and Peng [12] designed a model-free neural controller for handling the problem of friction force is function of local position and operation temperature. But it has about 0.5 mm error appearing at the acceleration discontinuous location. Wai et al. [13] proposed a robust fuzzy neural network controller to compensate the nonlinear friction behavior of this piezoelectric actuating table to reach 0.08mm accuracy. Mainali et al. [14] designed a hybrid sliding mode control and iterative learning control methods and then used to learn the friction disturbance repeatedly. Tan et al. [15] employed an exponential friction model to design

a learning nonlinear PID controller. These model-free approaches need long period of learning for improving the control accuracy. In order to improve the controller designing facility, a LuGre friction model is chosen and identified for this PZT actuating X-Y table for model-based adaptive controller design. Since the friction behavior of this experimental system has nonlinear and time-varying features, the functional approximation (FA) scheme [16] is employed to compensate the system modeling error of LuGre model. The control performances of a PID and the proposed adaptive control are compared based on experimental results.

II. THE EXPERIMENTAL SYSTEM STRUCTURE

A PC based controller is developed for this experimental system. This X-Y table has two independent axes x and y actuated by two different piezoelectric actuators. The experimental layout of this positioning system is shown in Fig. 1. This positioning table can be used in micro or/sub-micro level optical axis alignment of laser diode and lithography applications. The control voltage is calculated in PC, which is converted from digital to analog signal by a D/A interface card, and sent to the piezoelectric actuator's driver unit to actuate a piezoelectric motor. This X-Y table's displacements are measured by linear encoders, and sent back to PC through encoder card for closed loop control. The resolution of linear encoder is 0.1 μ m, and the maximum measuring range is 100mm. HR2 motor was used to actuate axis x, and HR4 motor was used to actuate axis y. If the offset control voltage used to compensate the friction dynamics is deducted from the control law, there has a linear response feature between the piezoelectric mechanism sliding velocity and the driver control voltage. The actuator and driver can be modeled as a DC-motor with internal friction, which is driven by a voltage amplifier, as Fig. 2. When a command voltage within ±10V is sent to the driver unit, the driver generates a 39.6KHz sinusoidal wave for driving the actuator with an amplitude function of the command voltage. This constant oscillation frequency is generated from the driver unit which was supplied by Nanomotion Ltd.

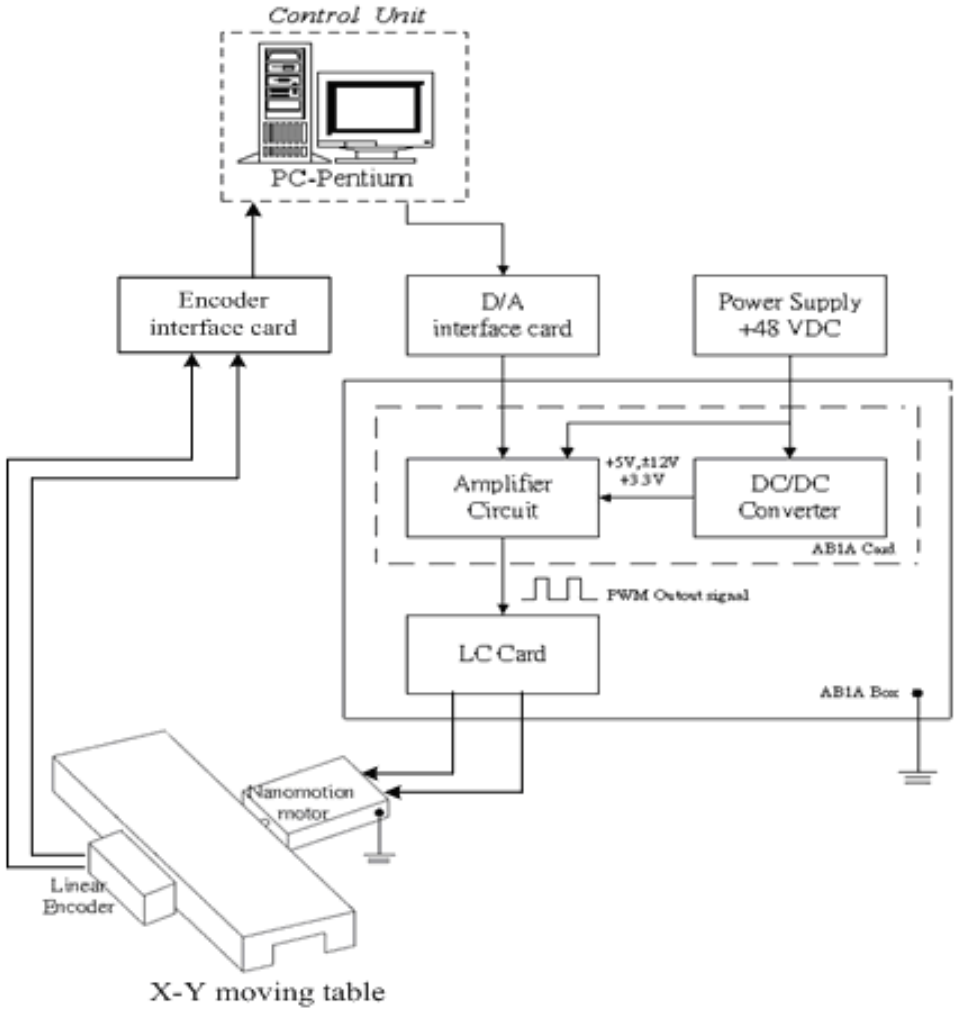


Fig. 1 Positioning control system experimental layout.

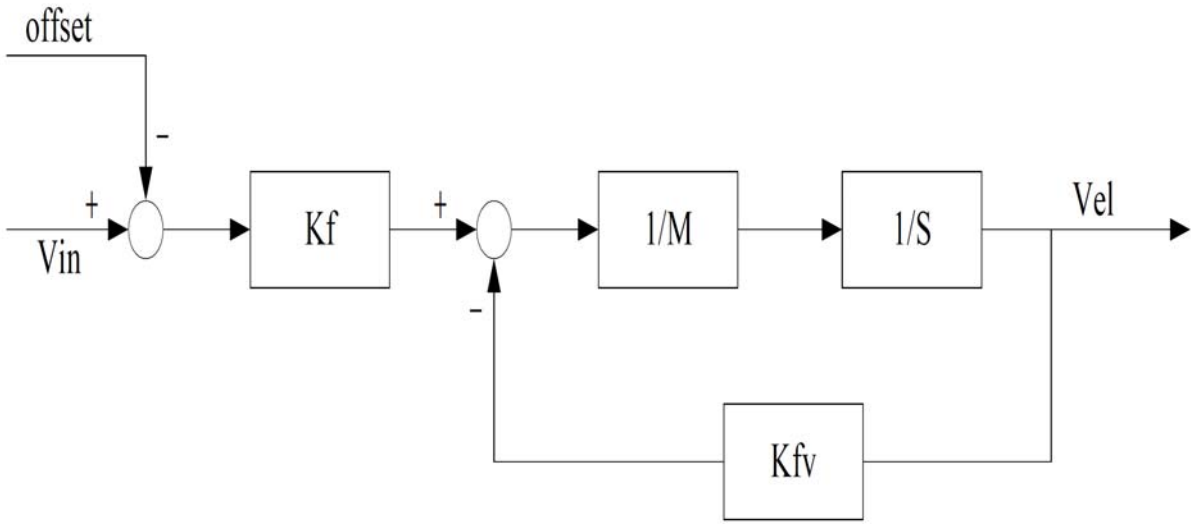


Fig. 2 The block diagram of motor and driver

where Vin is the command voltage of the driver (–10 ~ +10V) and offset is the dead zone voltage value (V). This offset voltage is a position dependent time-varying value which will be discussed in the next section. Kf is the force constant (N/V) and Kfv is the velocity damping factor (N×sec/m). Vel is the slide velocity (m/sec) and M is the moving mass (kg). S is the Laplace variable.

III. FRICTION MODEL OF THIS PIEZOELECTRIC ACTUATING TABLE

The main friction force components of this PZT actuating table can be divided into static friction and stricbeck effect viscous friction. If the control voltage is increased slowly from zero, the control voltage used to overcome the mechanism stick friction and preload can be found when it begins to move. This starting control voltage has nonlinear characteristic, and it is function of position, moving direction and temperature. It is called the dead-zone offset voltage. If the experimental payload is small comparing to the 36N nominal preload, and the specified moving speed is much less than the 250 mm/sec maximal allowable velocity, the linear inversely proportional behavior of actuating force with respect to velocity can be ignored. The slide motion speed will be linearly increased with respect to the control voltage increasing. Hence, this dead-zone stick friction control voltage can be measured by experimental testing. Fig. 3(a) and 4(b) are the measured motion speed with respect to control voltage of axes x and y, respectively. It can be found that the offset voltage of axis x is about 0.61V to 0.65V and –0.67V to –0.75V, and axis y is around 1.80V to 1.93V and –1.68V to –1.74V in both moving directions. When the actuating voltage exceeds the dead-zone offset voltage, the motion of X-Y table becomes very sensitive to the control voltage. If the actuating voltage exceeds the offset voltage, each 0.1V extra control voltage will cause about 7mm/sec velocity change in axis x, and about 4.6mm/sec velocity change in axis y, respectively. Hence, it can be concluded that most of the control voltage is used to overcome this dead-zone offset control voltage. Besides, the dead-zone offset voltage has time-varying characteristic, it may change with respect to the operating position, the temperature and humidity. How to deal with this offset compensation is the key factor of this PZT mechanism motion control investigation.

Actually, the dead-zone offset control voltage is not accurate enough to model the complicate friction dynamics of this PZT mechanism for designing a model-based precision controller. Hence, the LuGre friction model is adopted for this research, which includes stricbeck effect, hysteretic, spring-like characteristics for stiction, and varying break-away force. The mathematical equations of LuGre model can be described by bristle model as:

$$\frac{dz}{dt} = v - \frac{|v|}{g(v)} z \tag{1}$$

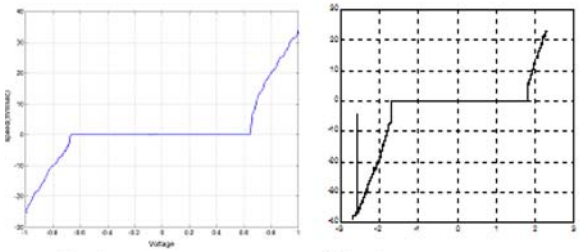
$$F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v \tag{2}$$

$$g(v) = F_c + (F_s - F_c) e^{-(v/v_s)^2} \tag{3}$$

Where v is velocity, z is the average deflection of the bristles, F is the friction force, σ_0 and σ_1 are the stiffness and damping coefficients, σ_2 is viscous friction coefficient. The function $g(v)$ is used to describe the stricbeck effect. v_s is the stricbeck velocity, F_c and F_s are the coulomb friction and static friction, respectively. Based on above equation, the steady state friction model can be derived.

$$F_{ss}=\sigma_0 g(v)\mathrm{sgn}(v)+\sigma_2 v=(F_c+(F_s-F_c)e^{-(v/v_s)^2})\mathrm{sgn}(v)+\sigma_2 v$$

(4)



(a) axis x (b) axis y

Fig. 3 Dead-zone offset voltage of real movement

The model parameters of this LuGre dynamic model, equation (1-3) and static LuGre model, eqation (4) can be identified based on experimental data by using nonlinear optimal operation scheme. Firstly, the steady state friction model is rewritten as:

$$F_{ss}=(\alpha_0+\alpha_1 e^{-(v/v_s)^2})\,\mathrm{sgn}(v)+\sigma_2 v$$

(5)

The parameters α_0 , α_1 , σ_2 and v_s can be estimated from the experimental friction-velocity curve. The friction force is calculated from the control voltage and the K_f (N/V) value of PZT data sheet. The constant velocity experimental data is obtained by designing a PI controller for speed control. The cost function is defined as:

$$\min_{\bar{\alpha}_0,\bar{\alpha}_1,\bar{\sigma}_2,\bar{v}_s}\sum_{i=1}^{n}[F_{ss}(v_i)-\hat{F}_{ss}(v_i)]^2$$

(6)

The function "fminunc"of Matlab optimization toolbox is used as the optimization scheme for estimating the steady state friction model parameters. For example, the parameters for the x direction axis are:

Positive velocity:

$\bar{\alpha}_0=0.6113;\bar{\alpha}_1=0.0699;\bar{\sigma}_2=0.0108;\bar{v}_s=0.1035$

Negative velocity:

$\bar{\alpha}_0=0.6337;\bar{\alpha}_1=0.1349;\bar{\sigma}_2=0.019;\bar{v}_s=0.1837$

IV. MODEL-BASED PID CONTROL AND ADAPTIVE FUCTIONAL APPROXIMATION COMPENSATION CONTROL

In order to investigate the feasibility of using this identified LuGre friction model for designing model-based controller, a sinusoidal wave trajectory is planned to evaluate the tracking control performance of a traditional model-based PID controller. The PZT system dynamics can be represented as:

$$m\ddot{x}=K_c(K_P e+K_D\dot{e}+K_I\sum e)-F_{friction}+d(x,t)$$

(7)

Where m and K_c are known mass and force/voltage gain of this PZT actuating system. $F_{friction}$ and $d(x,t)$ are the estimated LuGre friction force and unknown disturbance, respectively. The PID control gains are $K_P=10$, $K_D=0.007$ and $K_I=0.0002$. The dynamic response trajectory, tracking error and the friction force for tracking a 0.5Hz 1mm amplitude sinusoidal wave are shown in Fig. 4. It can be observed that the tracking error has obvious peak appearing in the motion direction (velocity) change. That means the LuGre model based PID control of this specific PZT friction actuating mechanism cannot correct the error on-time at these positions. Hence, the concept of Karnopp's model [1] is adopted to modify the LuGre friction model as:

$$F_{friction}=\begin{cases}-u & |\dot{x}|<\delta \quad |u|<f_{\max} \quad |e|>\phi\\ -f_{\max}shn(u) & |\dot{x}|<\delta \quad |u|\geq f_{\max} \quad |e|>\phi\\ LuGre\,\mathrm{model} & |e|<\phi\end{cases}$$

(8)

Where $f_{\max}$ and $u=K_c\hat{u}$ are the maximum static friction force and actuating force of the PZT actuator, respectively. The dynamic response trajectory, tracking error and the friction force for tracking a 0.5Hz 1mm amplitude sinusoidal wave by using the modified LuGre model are shown in Fig. 5. It can be observed that the tracking errors at the motion direction (velocity) change positions have been reduced significantly from 0.08mm to 0.03 mm.

Since this PZT friction actuating mechanism has large static friction and time-varying behavior, the LuGre model can not always accurately track this plant slow speed dynamics. Hence an adaptive functional approximation scheme is adopted to model this friction modeling error. The system dynamics can be written as

$$m\ddot{x}=K_c\hat{u}-\hat{F}_{friction}+\Delta F_{friction}+d(x,t)\equiv K_c\hat{u}-\hat{F}_{friction}+f(x,t)$$

(9)

If the control law is designed as

$$\hat{u}=\frac{1}{K_c}[m\ddot{x}_d+\hat{F}_{friction}-\hat{f}+k_0 e+k_1\dot{e}]$$

(10)

Where $e=x_d-x$ and $\hat{f}$ is the estimated value of the friction modeling error and dynamics uncertainty by using finite terms functional approximation function.

The unknown time-varying function $f(x,t)$ in equation (9) can be approximated by a linear combination of finite orthogonal basis functions $Z(t)$ to arbitrarily prescribed accuracy as long as n is larger enough [16].

$$\hat{f}(x,t)\approx \hat{W}_f^T Z_f(t) \qquad (11)$$

where $Z_f(t)$ is an orthogonal basis function vector and W_f is a weighting coefficient vector. If the number of basis functions is large enough, equation (11) can be described as the following approximation form.

$$f(X,t) = W_f^T Z_f(t) \tag{12}$$

where $Z_f(t) = [Z_1(t)\ \ Z_2(t)\ \cdots\ Z_n(t)]^T$, and $W_f = [W_1\ W_2\ \cdots\ W_n]^T$. This functional approximation equation (12) can be used to represent an unknown function with uncertainty and disturbance. The time-varying vector $Z_f(t)$ is a known function and the vector W_f is an unknown regulating constant. A proper Lyapunov function can be selected to find the update laws for these unknown constants based on Lyapunov stability theory.

The Lyapunov-like function is chosen as the summation of the square of system output error and parameters error ($\tilde{W}_f$).

$$V(E,\tilde{W}_f) = E^T P E + \tilde{W}_f^T Q_f \tilde{W}_f \tag{13}$$

Taking the time derivative of equation (13) and substituting the dynamic equations (9-10), the function parameters update law and Lyapunov-like function converging properties can be derived as;

$$\dot{\tilde{W}}_f = Q_f^{-1} Z_f b^T P E - \sigma \tilde{W}_f \tag{14}$$

$$\dot{V} = -E^T Q E \le 0 \tag{15}$$

Where $b^T = [0, K_c/m]$ is this dynamic system input gain vector and the right hand side second term of equation (14) is used to improve the parameter convergence property. That means this adaptive control system has globally stable property and the tracking error will be converged into certain error bound, which depends on the modeling accuracy.

V. EXPERIMENTAL RESULTS

In order to achieve a smooth motion response, the 3^{rd} order polynomial is chosen to plan the ideal moving trajectory between two positions.

$$L(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \tag{16}$$

The boundary conditions are

$$L(t_i) = L_i\ ,\ \dot{L}(t_i) = \dot{L}_i \tag{17}$$

$$L(t_f) = L_f\ ,\ \dot{L}(t_f) = \dot{L}_f \tag{18}$$

Then the coefficients of equation (4) can be solved as

$$a_0 = L_i\ ,\ a_1 = \dot{L}_i\ ,\ a_2 = \frac{3(L_f - L_i) - T(2\dot{L}_i + \dot{L}_f)}{T^2}\ ,$$

$$a_3 = \frac{-2(L_f - L_i) + T(\dot{L}_i + \dot{L}_f)}{T^3} \tag{19}$$

where L_i is the initial position of a trajectory, L_f is the final position of a trajectory, t_i is the initial time, t_f is the final time and $T = (t_f - t_i)$ is the motion time.

In previous literatures, the control objective of a micro-positioner is focused on the steady state accuracy of a step input response [17-18]. Usually, the transient response between each step change is not investigated for slow micro-stepping motion. If the moving speed and the step change are increased, the transient response should be concerned, too. Since this piezoelectric actuating table is driven by a PZT actuating finger tip instead of PZT directly, it has larger moving travel. Hence both the steady state accuracy and the transient state tracking error need be investigated in this study. In order to evaluate the control performances of the LuGre friction model based PID control and the modified LuGre model with adaptive functional approximation compensation controller, different moving speeds and travels are planned for experimental evaluation. The 400 Hz sampling frequency was chosen for the following experiments.

A. LuGre friction model based PID control

The appropriate control gains of this PID controller are chosen as $K_P = 10$, $K_D = 0.007$ and $K_I = 0.0002$ after a trial-and-error test. The modified LuGre friction model, eq.(8) is employed. The average velocity of a moving trajectory with 3^{rd} order polynomial is specified as 2mm/sec. The tracking response, error history and control voltage of x axis by using a PID control are shown in Fig. 6(a), 6(b) and 6(c), respectively. Although the final steady state error is less than $5\mu m$, the maximum middle lag is 18 μm, and the initial periods has a 12 μm oscillation due to the dead-zone offset control voltage chattering. In addition, the control gains need be searched by a trial-and-error process for each different motion situation to obtain reasonable control performance. It is not convenient for practical implementation. For example, the experimental results of tracking a 3^{rd} polynomial with 5mm/sec average speed by using this PID controller are shown in Fig. 7. It can be observed that the maximum middle lag is 70 μm, and the initial and settling periods have a 20 μm oscillation. The control voltage is smooth except at the initial starting area with moving directions change due to dead-zone offset control voltages.

B. Modified LuGre model with adaptive functional approximation compensation

The LuGre friction model, eq. (8) is employed and the adaptive functional approximation control parameters

are listed in Table 1. The tracking control experimental responses of this piezoelectric actuating table by using the proposed adaptive FAT controller of x axis are shown in Fig. 8 with 5 mm/sec average planning speeds.

It can be observed that the steady state error is less than $0.3\mu m$ and the initial learning period tracking error is less than 20 μm. The maximum middle lag is less than 10 μm. The control voltage chattering phenomenon only appears at initial learning and final stop positions. The proposed adaptive FAT LuGre friction model based controller can be employed in different moving speeds without trial-and-error effort for finding control parameters. The dynamic response and tracking error history of Y axis for tracking a sinusoidal wave with 1 mm amplitude are shown in Fig. 9. It can be observed that the tracking error always keep within 35 μm and the steady state error can reach 0.5 μm positioning accuracy. It can be concluded that the proposed adaptive FAT LuGre friction model based controller can effectively controlled this heavy friction and nonlinear PZT actuating mechanism to reach sub-micro level precision control accuracy.

It also can be observed from these experimental results that the steady state error can converge into 0.3μm finally for each case. In addition, the chattering phenomenon of control voltage is improved by using the modified hybrid Karnopp + LuGre friction.

VI. CONCLUSION

The LuGre friction model is chosen to model the friction dynamics of a piezoelectric actuating system. The concept of Karnopp's model is adopted to modify the LuGre friction model for model-based PID controller. The FAT is employed to design an adaptive control loop for compensating the modeling error and system dynamics uncertainty. The adaptive law is derived from the Lyapunov-like stability criterion to obtain the system converging property and stability. The tracking errors of a 3rd polynomial trajectory and a sinusoidal wave trajectory are kept within 20 μm and the final steady state error is less than 0.3 μm for both axes by using the proposed adaptive FAT hybrid LuGre model controller. These dynamics performances are better than that of a LuGre friction model based PID controller. In addition, the proposed adaptive controller can be employed to monitor different planning trajectory and speeds with the same control structure and parameters. In addition, the chattering phenomenon has been improved significantly by using the proposed adaptive controller.

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$A^T P + PA = -Q$	$A = \begin{bmatrix} 0 & 1 \\ -k_0 & -k_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -8/m & -0.004/m \end{bmatrix}$ $Q = \begin{bmatrix} 8\times10^4 & 0 \\ 0 & 1 \end{bmatrix}$
$\dot{\hat{W}}_f = Q_f^{-1} Z_f b^T PE - \sigma_f \hat{W}_f$	$Q_f^{-1} = 10 \cdot \mathrm{I}_{17\times17}$, $\sigma_f = 0.1$
$u = \frac{m}{K_c}[-\frac{\hat{f}}{m} + \ddot{x}_d - K^T E + \frac{F_{friction}}{m}]$	$m = 0.1425kg$, $K_c = 1$

Table 1 Adaptive functional approximation control parameters

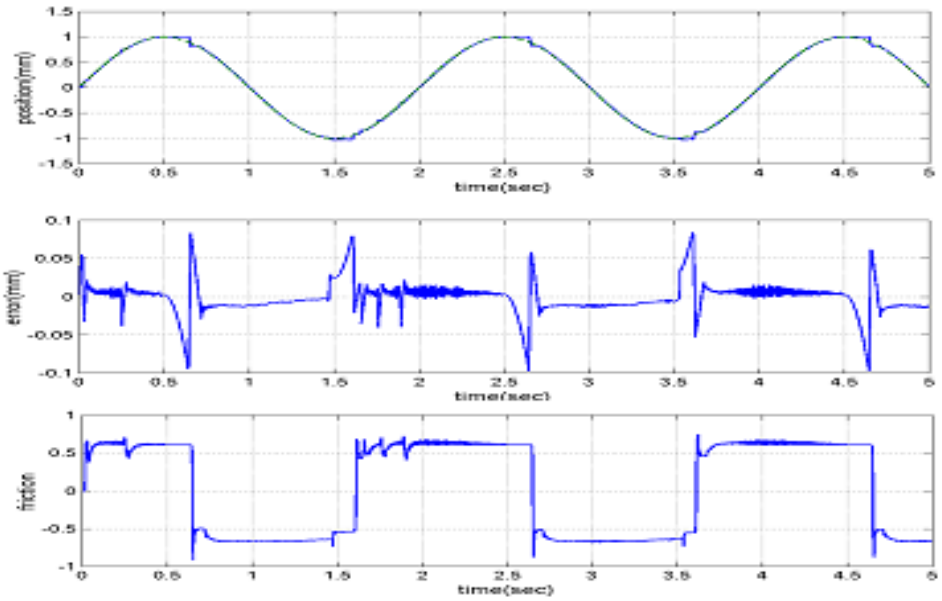


Fig. 4 The dynamic response trajectory, tracking error and the friction force for tracking a 0.5Hz 1mm amplitude sinusoidal wave by using PID control.

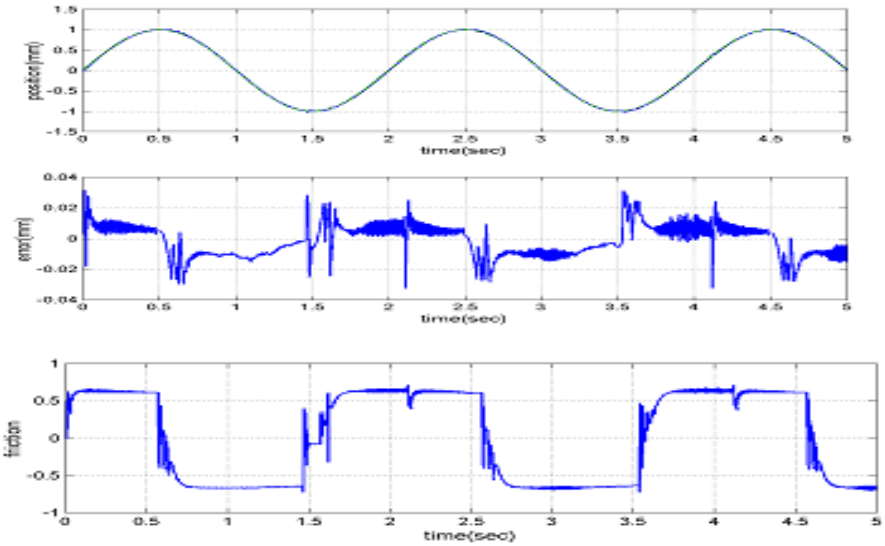


Fig. 5 The dynamic response trajectory, tracking error and the friction force for tracking a 0.5Hz 1mm amplitude sinusoidal wave based on modified LuGre model.

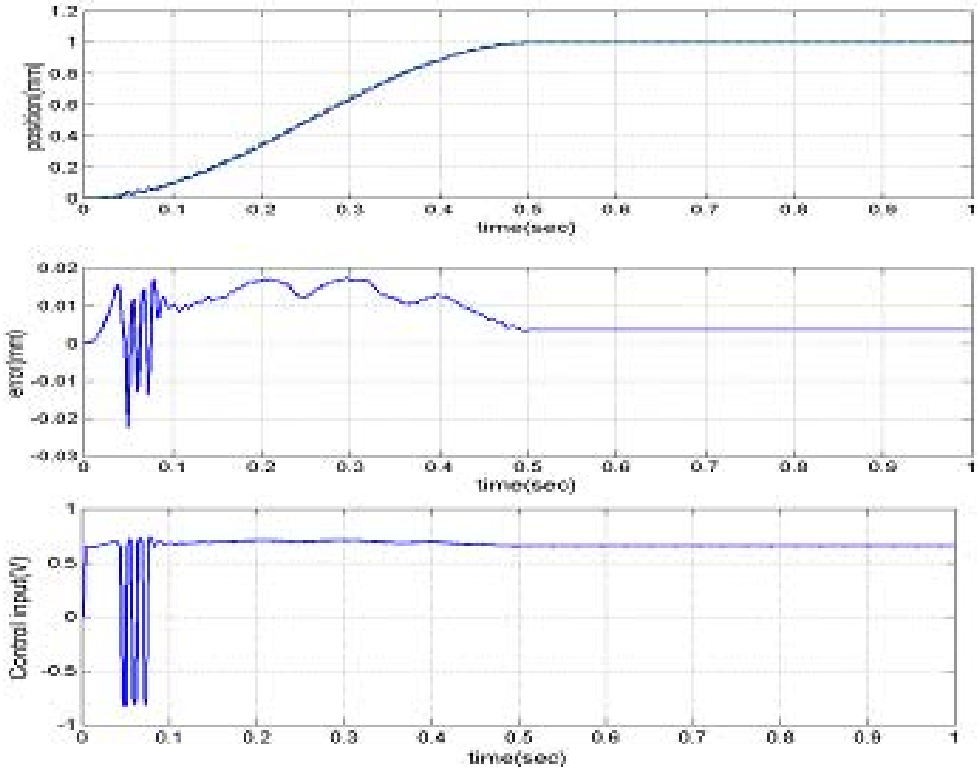


Fig. 6 The tracking response, error history and control voltage of axis x by using a PID control with 2 mm/sec average speed.

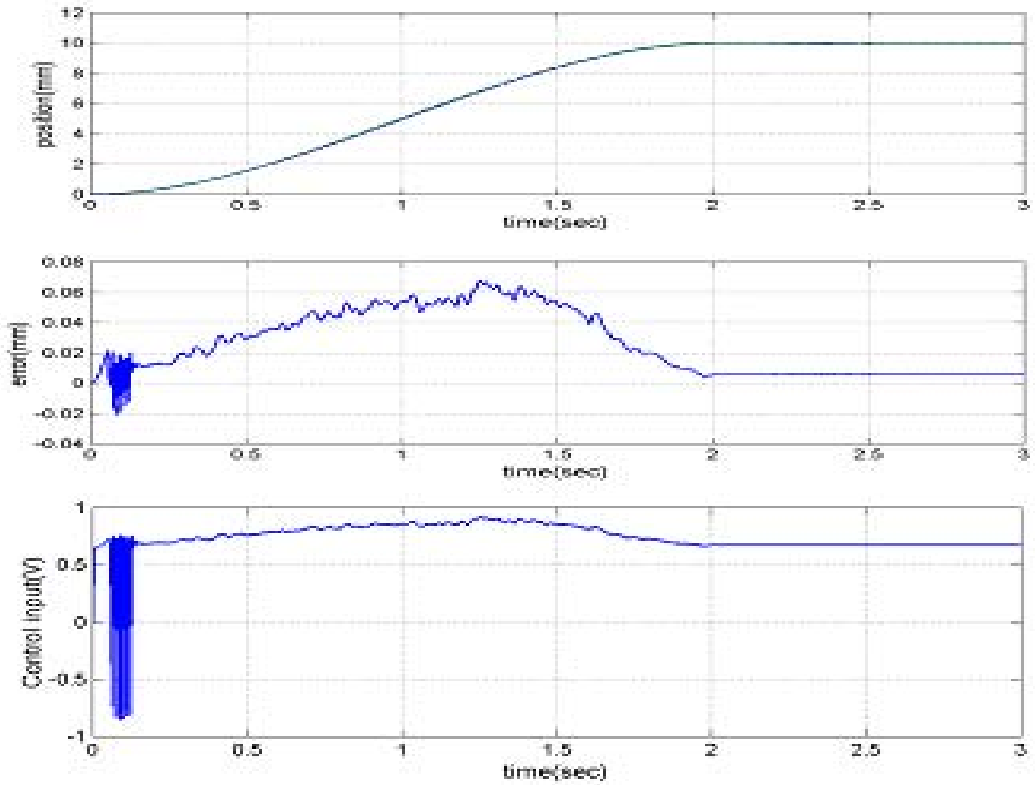


Fig. 7 The tracking response, error history and control voltage of axis x by using a PID control with 5 mm/sec average speed.

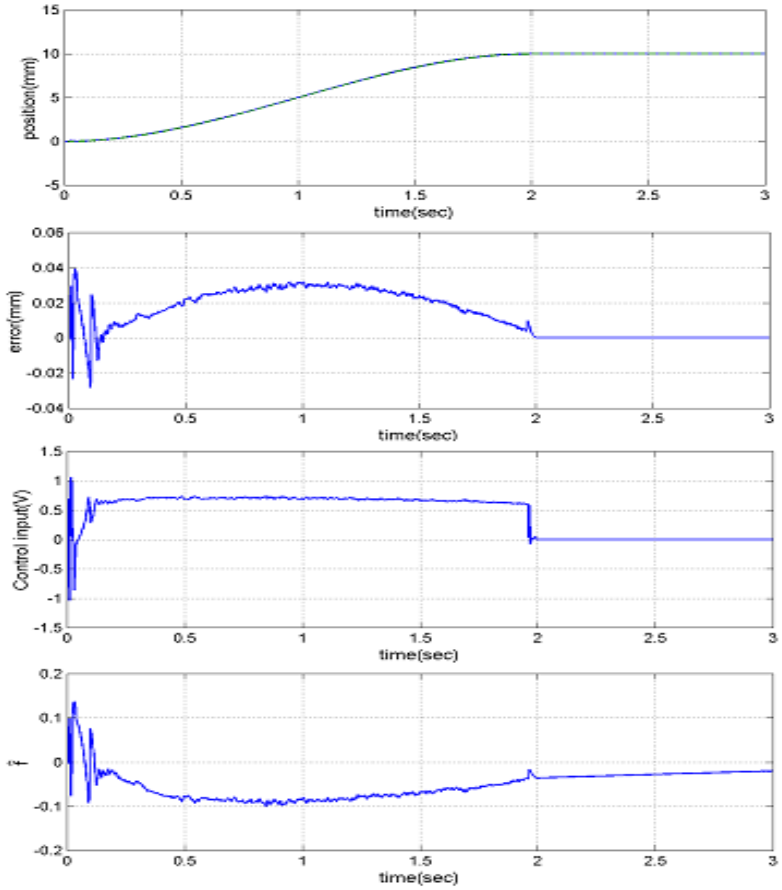


Fig. 8 The dynamic response, tracking error histories, control voltage, and estimated driving force for 5 mm/sec average speed.

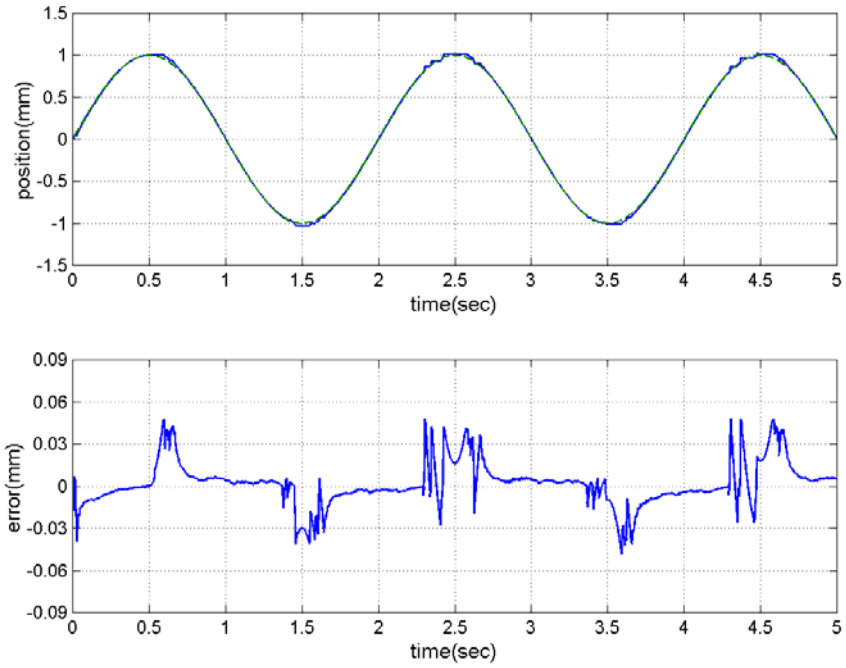


Fig. 9 Dynamic response and tracking error of y axis with 1 mm amplitude sinusoidal wave by using proposed adaptive FAT LuGre model compensation.

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