TESTING THE LIMIT CYCLE BEHAVIOR IN HOMOTHERMAL MAINTENANCE OF SKUNK CABBAGE

[1]TURTOGTOKH ERDENEBAATAR, [2]SHUJI KAWASAKI, [3]KIKUKATSU ITO, [4]KOJI MARUYAMA, [5]PUREVTSOGT NUGJGAR

[1,2]Graduate School of Science and Engineering, Iwate University, Morioka, Japan
[3]Graduate School of Agriculture Iwate University Morioka, Japan
[4]Graduate School of Science Osaka City University, Wolfram Research Asia Limited Osaka, Japan
[5]Graduate school of Engineering and Applied Sciences, National University of Mongolia Ulaanbaatar, Mongolia

Abstract - The wild plant called Skunk Cabbage has a heating organ and heats itself at the end of winter, to raise seeds effectively. The body temperature is well controlled so that it is retained almost constant at around 22~24[°C] for several days, even when the ambient temperature falls below freezing. The temperature control mechanism is considered to have a linear equilibrium phase and nonlinear nonequilibrium phase, according to the displacements of the body temperature from a target temperature called central temperature. In our previous study, we have shown that linear behavior is well modeled by certain mixed Poisson distribution, which considers the conflicts of actions of increasing or decreasing temperatures. In this study, we consider modeling the nonlinear behavior part which is report here applicability of so-called Van der Pol equation as a possible candidate model. As well as we did attempt to determine whether the limit cycle behavior exists.

Keywords - Nonlinear Nonequilibrium State, Limit Cycle Phenomena, Van der Pol Equation, Least-Squares Fitting, Chi-Square Goodness of Fit Test.

I. INTRODUCTION AND BACKGROUND

Skunk cabbage of the Araceae family is a perennial wildflower that sprouts in marshy, wet areas of forest lands and has the special feature of being equipped with a heating organ called a *spadix*. The organ generates heat continuously for several days before spring and retains its temperatures mostly constant at about 22~24 [°C], to nurture its seeds effectively [1-2].

As in the previous research [2], we have recognized that temperature control is something like a seesaw mechanism. That is, when the temperature gets lower than a particular value, the plant tries to raise it, and when the temperature gets higher than the value, the plant tries to lower it. We call the target temperature, toward which the plant tries to raise or lower the present temperatures in the next step, the central temperature (CT). The CT is unique for a plant sample and the CT values, denoted by x_0, are different over samples in general. Besides, the larger the temperature displacements from the CT be, the stronger the plant tries to move the temperatures toward the CT. The control is thus, in a sense, like a harmonic oscillator, or a spring, although the periodicity is not a constant. The linear behavior part is well elucidated by the elastic force of spring (see fig. 2).

Our data is the body temperature time series, denoted by $X(t)$, $t = 0,1,2, \ldots$, which is measured by a digital thermometer. We will consider only the first part of the time series. Because of the spadix temperature has roughly remained constant throughout the first part. The temperature data is measured once every minutes and the values are of 0.1 degree step, due to our measurement device. (see fig. 1)

The temperature control is considered as a result of the conflict of actions of temperature increasing and decreasing that are always taking place simultaneously. The circumstance of the conflicts is dependent on the present temperatures $X(t) = x$. The lower the temperature becomes, the stronger the backlash to return to CT becomes. And similarly, the higher the temperature becomes, the stronger the backlash to return to CT becomes. In another way, the larger the displacements $|x - x_0|$, the stronger the backlash becomes.

Fig. 1, every value of x is taken a lot of times in the time series. Therefore, we consider, as statistics of the increments from the present temperature x , taking the conditional expectation

$$y(t) = X(t+1) - X(t) \tag{1}$$

of the increments for each of the x. We remark that $y(t)$ may be assumed to be a 'stationary process'. Hence, we observe, as a characteristic of the control, the conditional expectation is

$$\varphi(x) = E[\, y(t) \mid X(t) = x] \tag{2}$$

and

$$\psi(x) = E[y(t+1) - y(t) \mid X(t) = x] \tag{3}$$

given by this. This $\varphi(x)$ may be considered a substitute for $\frac{dX}{dt}$. We also use $\psi(x)$ as the substitute for $\frac{d^2X}{dt^2}$.

We have been interested in its temperature control mechanism and found that it has a linear and nonlinear behavior region of temperatures. As for the linear region, we have clarified that homothermal maintenance is the result of the simultaneous occurrence of heat generation and absorption according to a mixed Poisson distribution. Also, the action of the control is well elucidated by the ordinary harmonic oscillation on average. On the other hand, outside of the linear region, there exists a nonlinear behavior region. (see fig. 2)

Then we found that the behavior of $\varphi(x)$ and $\psi(x)$ can be very well elucidated with the parameter values. Especially $\varphi(x)$ and $\psi(x)$ are both linear with the form

$-k(x - x_0)$ in the neighborhood of CT. The action of the linear part is similar to that of an elastic force of spring on average, according to our previous research [2].

Our aim was the modeling which indicates the time series complicated set has contained the phase transition model with a linear system of equilibrium state and a nonlinear system of nonequilibrium state. We analyze the time series data as in Figure 1, then we did consider curve fitting a time series model to the data. Then,

- How about the nonlinear part, which is outside of the CT-neighborhood?
- What kind of model of mechanism can elucidate the control mechanism?

After several trials, we arrived at the nonlinear dynamics of spring [3-4] when the displacements are large.

Although modeling of the mechanism in the nonlinear region is much harder than in linear region, we report here the possibility of a candidate of the model, called Van der Pol (VdP) equation. VdP found stable oscillations, now known as limit cycles, which is an isolated closed trajectory in the phase-plane corresponding to periodic motions [5-8].

II. MODELING BY VAN DER POL EQUATION

To model the homothermal maintenance mechanism including the nonlinear part as well, we consider applying a nonlinear dynamics of the elastic force of a spring, which is known as Van der Pol (VdP) equation [5-6]. The basic idea of the equation is as follows. First, the linear elastic force is given, as a dynamical system,

$$\begin{cases} \dfrac{du}{dt} = v, & \text{(4a)} \\ \dfrac{dv}{dt} = -ku & \text{(4b)}, \end{cases}$$

or, in the well-known form,

$$\frac{d^2u}{dt^2} = -ku \qquad (5)$$

Here, u is the displacement and v momentum. Adding the frictional force with frictional coefficient $\gamma_0 > 0$, we have a more realistic form

$$\frac{dv}{dt} = -\gamma_0 v - ku \qquad (4b)',$$

The VdP equation is the one that is made frictional coefficient occasionally varies to negative as well:

$$\frac{dv}{dt} = -\gamma v - ku, \gamma = \gamma_0(1 - u^2) \qquad (4b)'',$$

or, what is the same

$$\frac{d^2u}{dt^2} = -\gamma \frac{du}{dt} - ku \qquad (5)''.$$

When the frictional coefficient γ is negative, the system receives energy from the aether force, and it performs work on the system.

Let us observe our data for the 1st order derivative $\frac{dX}{dt}$ in Figure 3a and 3b. How does it look like? In addition to the linear term which has been incorporated in the study of linear part already, it may seem to have another, nonlinear term with a negative coefficient of anti-symmetry. Among such a term, one of the simplest may be given by x^3. Thus, for $\frac{dX}{dt} = \varphi(x)$, we consider taking a candidate model $\varphi(x) \cong -a_1(x - x_0) - b_1(x - x_0)^3$, or,

$$\varphi(u) \cong -a_1 u - b_1 u^3, \qquad (6)$$

using the same symbol $\varphi(x)$ and putting $x - x_0 = u$. If this is the case, then we have

$$\frac{du}{dt} = v, \qquad v = -(a_1 u + b_1 u^3) \qquad (7a),$$

$$\frac{dv}{dt} = \frac{dv}{du} \cdot \frac{du}{dt} = -(a_1 + b_1 u^2) \cdot v \qquad (7b),$$

i.e.

$$\frac{d^2u}{dt^2} = -(a_2 + b_2 u^2) \cdot \frac{du}{dt} - ku \qquad (5)'''$$

which is similar to (5'').

III. FITTING VdP TO DATA

We first checked out that $\varphi(x)$, the substitute for the first-order derivative $\frac{du}{dt}$, has indeed the first and third-order terms of displacements $x - x_0$, which is also the substitute for u. In order for that, we set a model equation for $\varphi(x)$ as

$$f_1(x) = a_1(x - x_0) + b_1(x - x_0)^{p_1} \qquad (8)$$

for a_1, b_1, $x_0 \in \mathbb{R}$ and $p_1 > 0$, and estimated the parameters that makes $f_1(x)$ fit best, expecting p_1 would be close to 3.

For the model equation of (7a), we fitted a function $f_1(x)$. There, we attach coefficients a_1, b_1, x_0 and p_1 for the 1st order term and 3rd order term, respectively. As a result of the fitting, we have estimated parameter values as in Table 1. In addition to this, we can see that it is indeed close to 3 (see Table II), so that we may consider that it has the first and third-order terms as in (7a). To calculate, we have used least-square curve fitting and performed the fitting to make the least square error is smaller than 5% and that absolute value is taken over all points of x in the interval. Such good fitting is obtained for twelve samples, eleven have errors. (see fig. 3a and 3b)

Next, using the estimated $f(x)$ as the expression of $\frac{du}{dt}$, we checked out that $\psi(x)$, for $\frac{d^2u}{dt^2}$ We thus consider

$$f_2(x) = [a_2 + b_2(x - x_0)^{p_2}]f_1(x) + k(x - x_0) \qquad (9)$$

and estimate $a_2, b_2, k, x_0 \in \mathbb{R}$ and $p_2 > 0$, to have an optimal fitting of $f_2(x)$ to the model equation of the form (9). The estimated the parameters that makes $f_2(x)$ fit best, expecting p_2 would be close to 2.

For the model equation of VdP equation, we fitted a function $f_2(x)$. There, we attach coefficients a_2, b_2, k, x_0 and p_2 for the 1st and 2nd order term, respectively. As a result of the fitting, we have estimated parameter values as in Table 2. We can see that it is indeed close to 2 (see Table III), so that we may consider it as in VdP equation. Also, such good fitting is obtained for twelve samples, eleven have errors. (see fig. 4a and 4b)

As well in $f_2(x)$, a_2, b_2, and k are estimated to be negative. In these fittings, the frictional coefficients a_2, b_2 being negative implies the system embraces negative 'resistance' so that it is indeed in a nonequilibrium state receiving energy from external forces. The system of VdP with the negative 'resistance', is known to show the so-called limit cycle behavior. On the other way, if, in VdP equation

$$\frac{d^2u}{dt^2} = -(a_2 + b_2u^2) \cdot \frac{du}{dt} - ku$$

the frictional coefficient of

$$a_2 + b_2u^2 < 0$$

is negative, i.e. then the system has a 'negative resistance', in general. Then all of our samples turn out to be the limit cycle (see fig. 5a and 5b). In such cases, it is known that the orbit of the solution is just circular around the origin in (u, v) phase plane. These figures are the orbits of our data. These orbits may be indicating that u and v go around the origin.

Although a more detailed verification is necessary for the limit cycle, we think this may be suggesting the limit cycle [8-9] phenomenon in the homothermal maintenance: By going around the origin for a long time, the plant is trying the temperatures and its derivative to stay in a small range. We guess this is the basic mechanism of homothermal maintenance.

Hypothesis testing for the curve fitting

After curve fitting data with the model, we need to evaluate the goodness of fit.

For the tests of normality, we use the Shapiro-Wilk parametric hypothesis test of composite normality. That test to determine if the null hypothesis of composite normality is a reasonable assumption regarding the population distribution of sample expectations and fit curve. The desired significance level, $\alpha = 0.01$, and $\alpha = 0.05$. For each value of delta, we examined the normality with deltas that result in good exponent values in the fitting. Hence, most of the results indicate that the dataset is normally distributed [10]. (see fig. 6 and Table I)

In this section, we perform the hypothesis testing for the curve fitting [11-12]. We use the Chi-square Goodness of Fit Test. First, Clearly, state the null (H_0) and alternative H_A hypotheses.

H_0: there is no significant difference between the sample expectations and the fit curve.

H_A: there is a significant difference between them.

Chi-Square Goodness of fit test is appropriate because we are examining the distribution of a single categorical variable. We assume that the significance level is 0.05 in the problem.

The degrees of freedom, therefore, is $k - 3$ (*normal distribution*). To calculate our Chi-square statistic, we need to sum the squared difference between each sample expectation value and fit curve value divided by the fit curve value:

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} \qquad (10)$$

Now that we have our Chi-square statistic, we need to compare it to the chi-square value for the significance level 0.05. We can use a reference table. Using the table, we usually find that the critical value for a significance level with $df = 55$ and 60, etc. is 38.96 and 43.19. On the other hand, our chi-square statistic result is less than critical value, so we will not reject the null hypothesis. Since our chi-square statistic was less than the critical value, we do not reject the null hypothesis, and we can say that there is no significant difference between the sample expectation and fit curve.

IV. CONCLUSION

In this research, we developed a transition model with a linear system of equilibrium state and a nonlinear system of nonequilibrium state for the Skunk cabbage. Our ultimate goal was showing the Van der Pol equation where an occurs limit cycle phenomenon in the nonlinear equation, based on curve fitting. The VdP equation might be formally a candidate model of the dynamics of the homeothermal maintenance mechanism.

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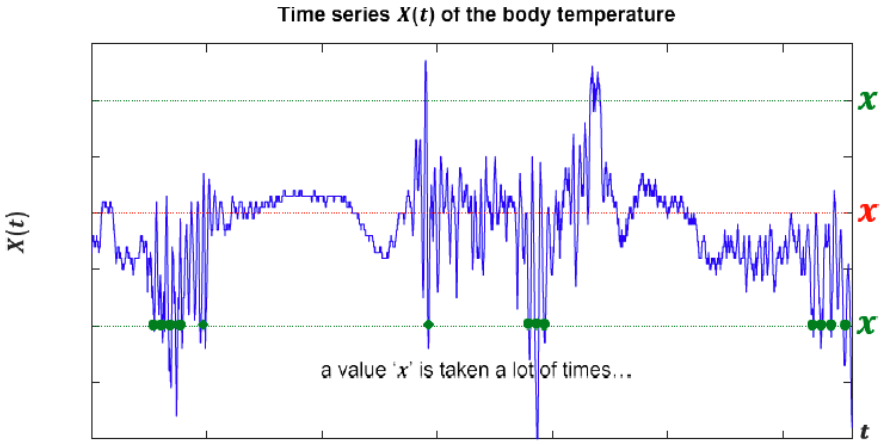


Figure 1. Time series X(t) of the body temperature. Here is the first part of the sample time series. The blue spiral is the body temperature of the plant and the red line is the central temperature.

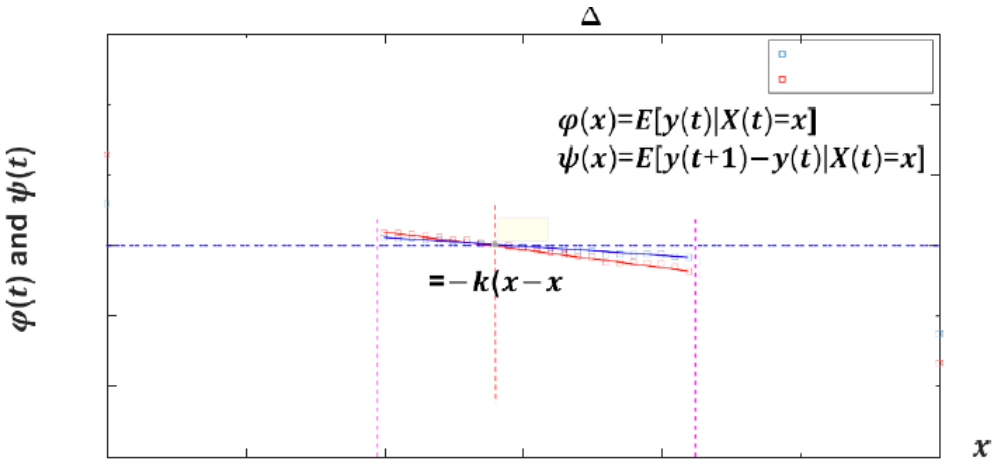


Figure 2. The fitting for 1st and 2nd order derivative. In this figure, to see the dependence of the mean of φ(x) and ψ(x) on x, we took the plot of the mean over x. The blue and red squares are the mean of the actual data, and red and blue lines are a linear equation.

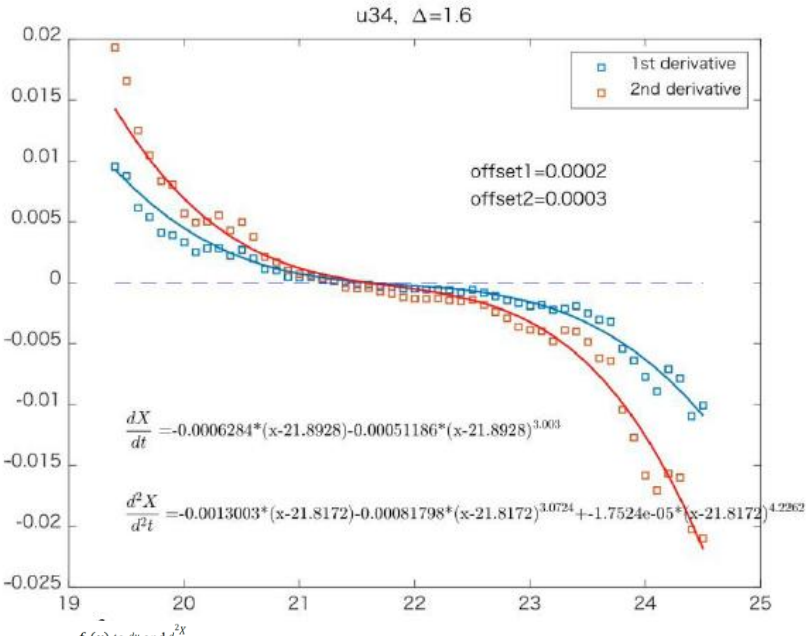


Figure 3a. Fitting the curve $f_1(x)$ to $\frac{du}{dt}$ and $\frac{d^2X}{dt^2}$. Here, the red and blue square is the mean of the real data, and a red and blue line is a fitting curve of $f(x)_1$ function for the first and second-order derivatives. The fit seems apparently pretty good.

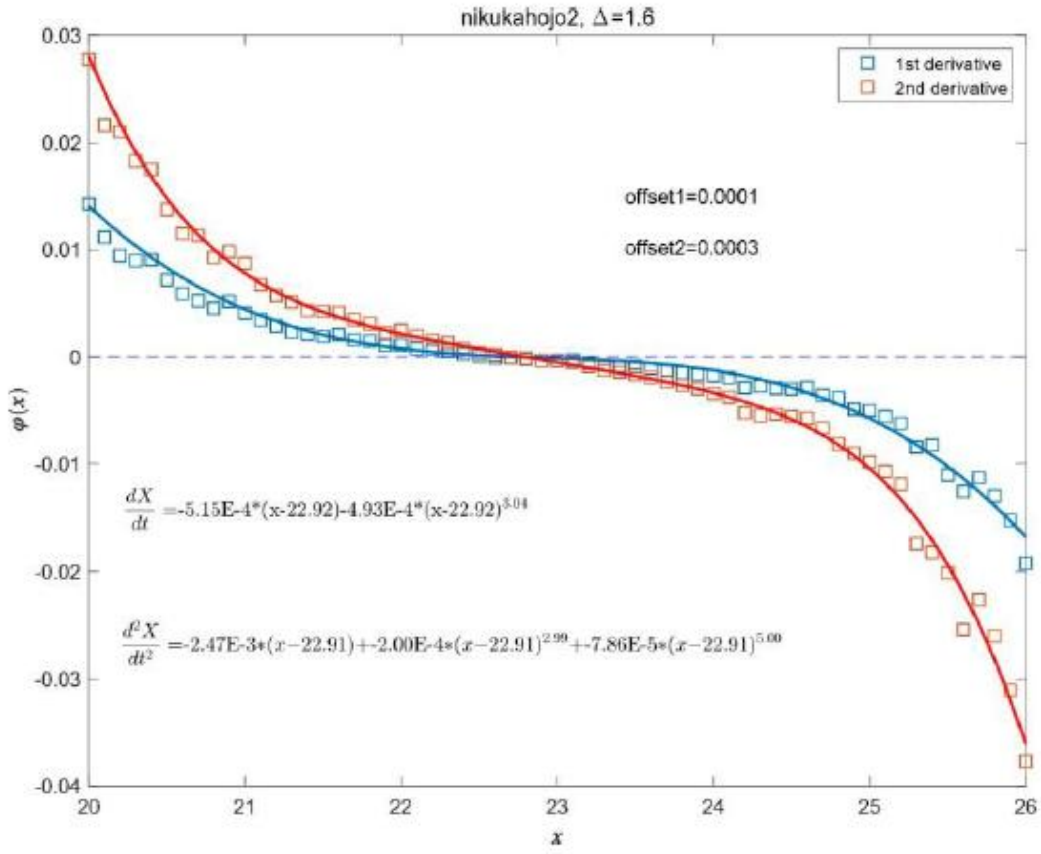


Figure 3b. Fitting the curve $f_1(x)$ to $\frac{du}{dt}$ and $\frac{d^2X}{dt^2}$. This is another sample and result is similar.

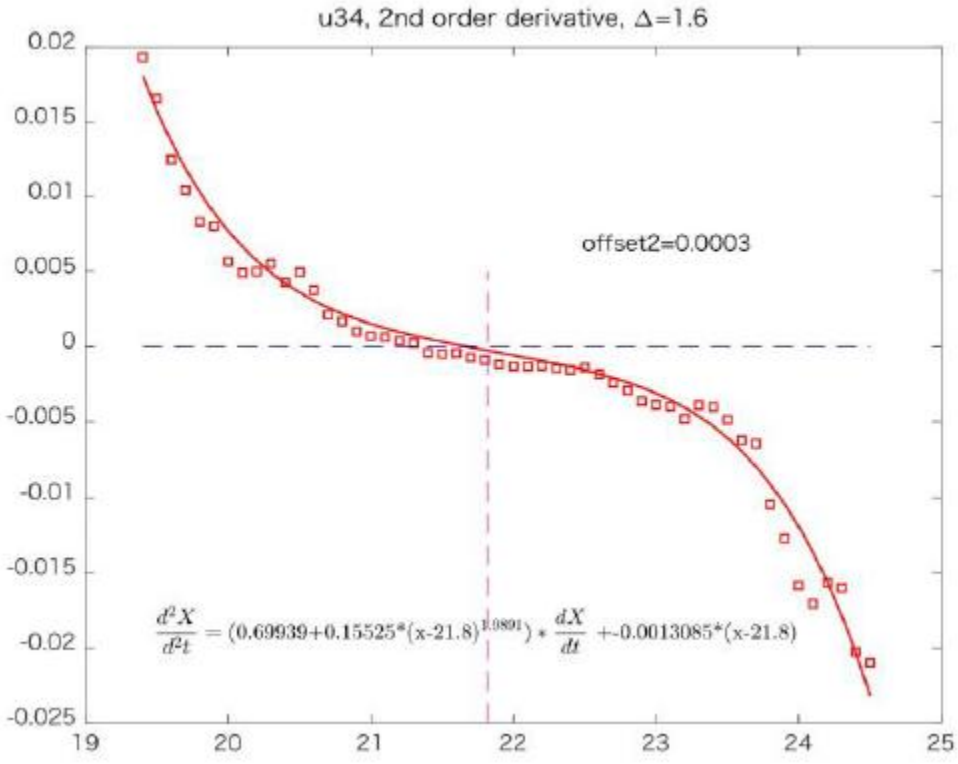


Figure 4a. Fitting the curve $f_2(x)$ to $\frac{d^2u}{dt^2}$. In this figure, the red square is the mean of the real data, and a blue line is a fitting curve of f2(x) function for the second-order derivative. Here as well, the fit seems very good.

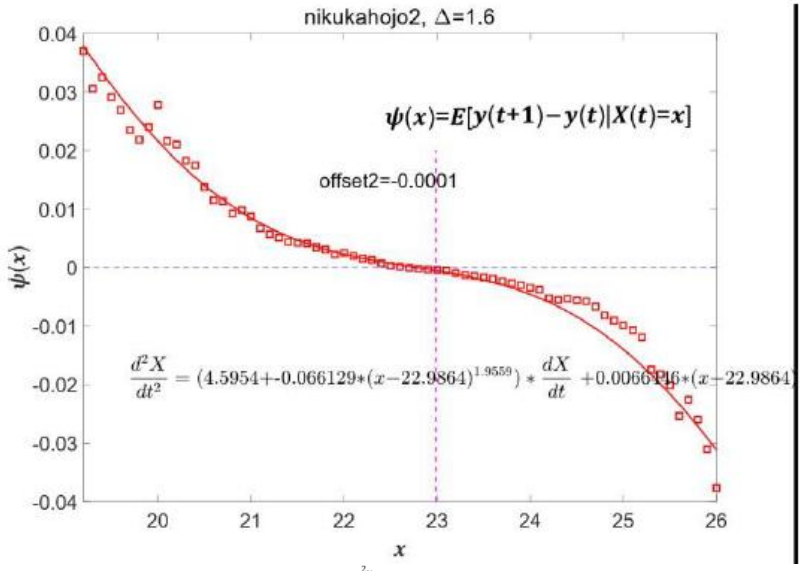


Figure 4b. Fitting the curve $f_2(x)$ to $\frac{d^2u}{dt^2}$. It is another sample and result is similar.

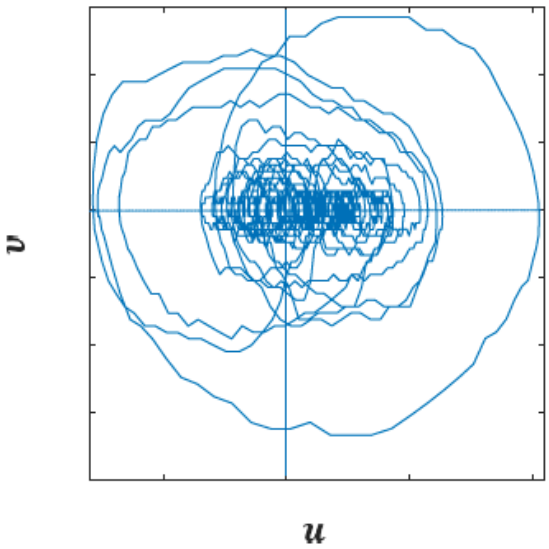


Figure 5a. Limit cycle for the VdP oscillator

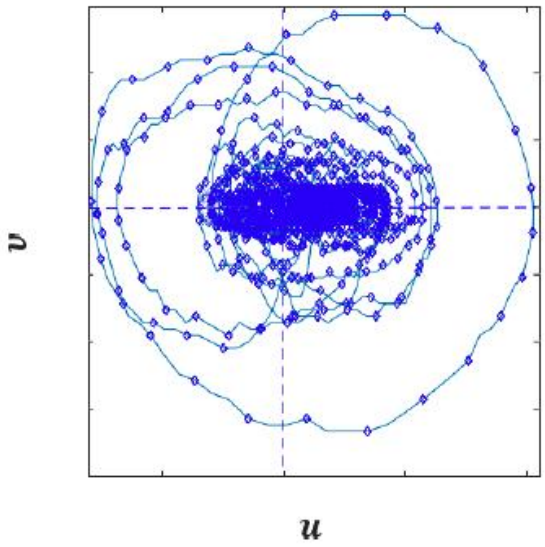


Figure 5b. Limit cycle for the VdP oscillator with animation

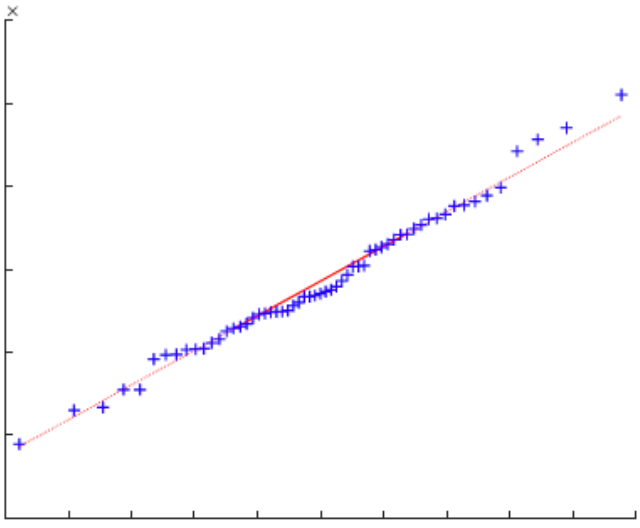


Figure 6. QQ plot of sample data versus standard normal.

Delta	H005	H001	pValue	stat
1.0	1*	0*	0.0416	0.9577
1.1	0	0	0.3748	0.9798
1.2	0	0	0.2600	0.9761
1.3	0	0	0.1370	0.9414
1.4	0	0	0.1161	0.9628
1.5	0	0	0.8450	0.9879
1.6	0	0	0.4872	0.9821
1.7	0	0	0.6491	0.9842
1.8	0	0	0.0876	0.9642
1.9	1	1	0.0049	0.9338

*H = 0 => Do not reject the null hypothesis at significance level.
**H = 1 => Reject the null hypothesis at significance level.

TABLE I: PARAMETER ESTIMATION

Sample No.	a_1	b_1	x_0	$\boldsymbol{p_1}$
1	$-6.23E-4$	$-5.12E-4$	21.9	**3.00**
2	$-5.15E-4$	$-4.93E-4$	22.9	**3.94**
3	$-1.56E-4$	$-1.40E-4$	22.9	**2.98**
4	$-4.15E-4$	$-8.34E-4$	23.6	**3.03**
5	$-1.20E-4$	$-3.84E-4$	23.3	**3.05**

TABLE II: PARAMETER ESTIMATION

Sample No.	a_2	b_2	k	x_0	$\boldsymbol{p_2}$
1	0.70	0.16	$-1.13E-3$	21.8	**1.99**
2	4.59	-0.07	$6.65E-3$	22.9	**1.96**
3	-0.88	-0.14	$-1.22E-3$	22.2	**2.01**
4	-0.34	-0.24	$-1.83E-3$	21.8	**1.97**
5	-0.39	-0.29	$-5.64E-4$	22.2	**2.05**

TABLE III: PARAMETER ESTIMATION

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