QUASI-STEADY AEROELASTIC ANALYSIS OF A DOUBLE SPRING PLATE FOR PROTOTYPE DESIGN

[1]ALFONSO URIBE, [2]DIEGO F. LEDEZMA, [3]EMILIANO RUSTIGHI

[1,2]Facultad de Ingeniería Mecánica y Eléctrica, Universidad Autónoma de Nuevo León, México
[1,3]Institute of Sound and Vibration Research, University of Southampton, UK

Abstract - The design of prototypes for aeroelasticity experiments without any experimental data or real structure is complicated. Many variables are used to stablish a model for analytical purposes, such as the values of the spring's constants,the aerodynamic model to use, and the location of the elastic, aerodynamic and gravity center. From the quasi-steady model, an aerodynamic plate model is stablished with a double spring arrangement. From a mathematical analysis, analyticalresults are shown.It is observed the change of the instability speed with parameters values either for flutter or divergence.

Keywords - Aeroelasticity, Divergence, Flutter, Quasi-Steady

I. INTRODUCTION

One of the most important design objectives in structural design is to reduce the weight while maintaining the strength to resist the stresses caused by the applied loads. This reduction has the consequence of reducing the overall stiffness of the structure and makes it susceptible to other types of failures. Failure by flutter is categorized as structural instability by wind-induced loads and it is an important topic in aeroelasticity dynamics.The failure develops when two natural frequencies get closer as the airspeed increases and the effective damping becomes negative. Increasing flutter speed orcompletelysuppressing it is the main goal when designing a passive, active or semi-active control method [1]-[2].For passive techniques, Verstraelen implemented the use of passive linear and nonlinear absorbers to control the non-linearities of flutter [3]-[4]-[5]. These results showedan increase of linear flutter speed. Furthermore, Lacarbonara and Cetrato analyzed the effect of using visco-hysteretic absorbers[6] where the hysteresis increased the rate of decay. The use of thin plates in aerodynamics and aeroelasticity simplifies the need to obtain data for airfoils with thickness/chord ratio below 12% using thin-airfoil theory [7]. Verstraelen demonstrated the use of the passive absorbers on a flat plate before utilizing a wing [3]-[4] and Pigolotti and Abu-Abduo experimented with different values of damping in a flat plate to see the effect on flutter [8]. The simplified model utilized for aeroelasticity isthe axial and torsional spring combination [1].

The location of both springs determines the elastic center, where a force can be applied without creating rotation in the wing. Other important locations are the aerodynamic center, where the lift, drag and aerodynamic moment are localized; and the center of gravity.Unless there is a real structure as a base, the design of an experiment is a complicated process to do because of the number of parameters to control and obtaining a desired failure. Also, the inclusion of bending changes the structure from a single degree of freedom to a continuum structure with multiple degrees of freedom. The goal of this article is to propose a simplified model to change from an axial and torsional combination to a two axial spring combination. This results in having a rigid body reduced to atwo-dimensional plane analysis instead of a flexible body.Depending of the parameters the springs, an equivalent axial and torsional spring can be obtained, and the elastic center can also be calculated. The purpose is tofacilitatethe process of designing a prototype where flutter can be achieved under wind tunnel restrictions without any real model to base on or, in another words, starting from zero. In the end, the goal of the prototype is to design and test a semi-active dynamic absorber, using magnetorheological elastomers, and compare it with its passive counterpart.

II. AEROELASTIC MODEL

The general equation to analyze an aeroelastic system [9] is

$$A\ddot{q} + (\rho vB + D)\dot{q} + (\rho v^2 C + E)q = 0 \quad (1)$$

where qis the coordinate vector, A is the inertial matrix, B and C are the aerodynamic damping and stiffness; Dand E are the structural damping and stiffness, ρ is the density of air in kg/m^3 and v is the airspeed in m/s. As a linear system, the system can be solved transforming (1) into a state-space form

$$\begin{Bmatrix} \dot{q} \\ \ddot{q} \end{Bmatrix} - \begin{bmatrix} 0 & I \\ -A^{-1}(\rho v^2 C + E & -A^{-1}(\rho vB + D \end{bmatrix} \begin{Bmatrix} q \\ \dot{q} \end{Bmatrix} = 0 \quad (2)$$

which also has the form of

$$\dot{x} - Qx = 0 \quad (3)$$

Eacheigenvalue of the matrix Qrepresents the modeof each coordinate with a frequency and a damping ratio

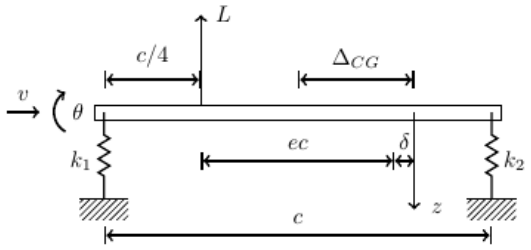


Figure 1: Plate Model with the location and direction of the lift force, z and θ

$$\lambda_j = -\zeta_j \omega_j \pm i\omega_j \sqrt{1-\zeta_j^2} \quad (4)$$

where λ_j is the eigenvalue, ω_j is the natural frequency in rad/s and ζ_j is the damping ratio of the coordinate j.Flutter occurs when the real part of a complex$\lambda_j$becomes positive or, in other words, when ζ_j is negative. Otherwise, if$\lambda_j$is real and positive then the system is unstable because of static divergence. The positive valuesare caused by a negative stiffness in the system.The model to analyze is shown in Fig. 1using z and θ as coordinates.

A. Aerodynamic Properties

For the aerodynamic model, the next considerations are used:

- The drag force is ignored since it is very small compared to the lift force.
- The section is symmetrical so there is no lift at zero angle of attack and no aerodynamic moment.
- Strip theory will be used to calculate the net lift force of the system.
- The quasi-steady consideration is assumedwhich means the value of the aerodynamic forces depends only of the instantaneous angle of attack. This does not include the aeroelastic derivatives.

The liftcaused by an airfoil section can be calculated utilizing the next equation

$$L' = \frac{\rho v^2 c c_l}{2} \quad (5)$$

Where L' is the lift forceper unit span in N/m, c is the chord of the airfoil in m and c_l is the lift coefficient of the airfoil that depends of the angle of attack α. According to thin airfoil theory,$c_l=2\pi\alpha$and the force is located at a quarter of the chord.Strip theory assumes a rectangular distribution over the wing, so the total lift of the wing is

$$L = sL' = \rho v^2 S\alpha \quad (6)$$

Where S is the surface of the plate in m^2 and L is the lift force generated by the plate. The value of α can be changed so it depends on the coordinates from the system.The angle of attack is the relative angle between the chord and the vector airspeed. If there is a rotation θ and/or a plunge speed $\dot{z}_{ec}$, located in the elastic center, then an angle of attack different to zero is obtained. If the angles are small, the total angle of attack is the sum of components shown in Fig. 2

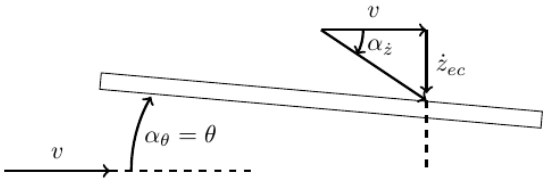


Figure 2: Angle of attack caused by θand $\dot{z}_{ec}$

$$\tan\alpha_{\dot{z}} = \frac{\dot{z}_{ec}}{v} \approx \alpha_{\dot{z}} \quad (8)$$

$$\alpha = \alpha_\theta + \alpha_{\dot{z}} \approx \theta + \frac{\dot{z_{ec}}}{v} \quad (9)$$

Combining the result of (9) with (6), the total L caused by the coordinates is

$$L = \pi\rho v^2 S\left(\theta + \frac{\dot{z}_{ec}}{v}\right) \quad (10)$$

which is necessary to calculate B and C for the vertical translation and rotation of the plate.

B. Structural Properties

As a simplified model, we can use Newtons laws to calculate each of the forces and moments caused by the springs. Depending of the combination of springs, the elastic center changes position. By using a sum of moments, the position from the trailing edge with a value P, shown in Fig. 3, is

$$P = \frac{k_2}{k_1+k_2} c \quad (11)$$

The dynamic equations will be generalized, which means that the coordinate z can be any point outside the elastic center. Using Fig. 1 and Fig.3as reference for the positions, the sum of forces and moments is

$$\Sigma F_{Pos_z} = m(\ddot{z} - \Delta_{cg}\ddot{\theta}) = -k_1(z - [P+\delta]\theta) - k_2(z + [c - P - \delta]\theta) - L \quad (12)$$

$$\Sigma M_{Pos_z} = J_z\ddot{\theta} - m\Delta_{cg}\ddot{z} = k_1(z - [P+\delta]\theta)[P + \delta] - k_2(z + [c - P - \delta]\theta)[c - P - \delta] + L[ec + \delta] \quad (13)$$

Where m is the mass of the plate in kg,J_z is the mass inertia moment at the point in the coordinate z in kg-m^2,Δ_{cg} is the distance between the center of gravity and the coordinate z,ec is the distance between de aerodynamic center and the elastic center; and δis the

distance between the elastic center and the coordinate z.By substituting (10) in (12) and (13); and rearranging the equations in the form of (1), the values of the matrices A, D and E, given by the structural properties; and the values of the matricesB andC, given by the aerodynamic properties, are

$$A = \begin{bmatrix} m & -m\Delta_{cg} \\ -m\Delta_{cg} & J_z \end{bmatrix} \quad (14)$$

$$B = \begin{bmatrix} \pi S & -\pi S\delta \\ -\pi S(ec+\delta) & \pi S\delta(ec+\delta) \end{bmatrix} \quad (15)$$

$$C = \begin{bmatrix} 0 & \pi S \\ 0 & -\pi S(ec+\delta) \end{bmatrix} \quad (16)$$

$$D = 0 \quad (17)$$

$$\mathbf{E} = [(k_1 + k_2\& - k_1\,(P+\delta) + k_2\,(c-P-\delta)@ - k_1\,(P+\delta) + k_2\,(c-P-\delta)\&k_1\,(P+\delta)\text{^}2 + +k_2\,(c-P-\delta)\text{^}2\,)] \quad (18)$$

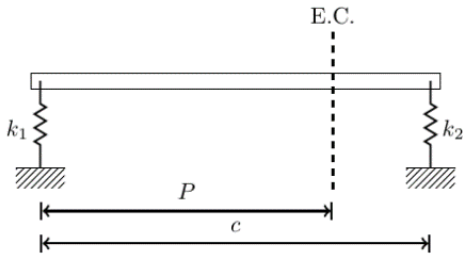


Figure 3: Location of the Elastic Center

The aerodynamic matrices (15) and (16) show a non-symmetric value so the eigenvalues will be always be a complex value. Also, matrix (16) has a negative term in the diagonal which, by the increase of v, leads to flutter.

III. SPECIAL COMBINATIONS

Depending on the position of the coordinate z, elastic center and center of gravity, the values of (14) to (18) change accordingly.

A. Coordinate zat the Elastic Center (δ=0)

By establishing both points at the same place, which is the classic way in aeroelasticity, then (16), (17) and (19) change into

$$B = \begin{bmatrix} \pi S & 0 \\ -\pi Sec & 0 \end{bmatrix} \quad (19)$$

$$C = \begin{bmatrix} 0 & \pi S \\ 0 & -\pi Sec \end{bmatrix} \quad (20)$$

$$E = \begin{bmatrix} k_1 + k_2 & 0 \\ 0 & k_1 P^2 + k_2(c-P)^2 \end{bmatrix} \quad (21)$$

The result in (21) shows a structural elastic uncoupling andthe natural frequencies of the system, at v=0, can be calculated by

$$\omega_j = \sqrt{E_{jj}/A_{jj}} \quad (22)$$

B. Elastic Center and Coordinate z at theCenter of the Plate (k_1=k_2)

If both springs have the same values, k_1=k_2=k, then the elastic center is located at the center of the plate and the values of (19), (20) and (21)simplify to

$$B = \begin{bmatrix} \pi S & 0 \\ -\pi Sb/2 & 0 \end{bmatrix} \quad (23)$$

$$C = \begin{bmatrix} 0 & \pi S \\ 0 & -\pi Sb/2 \end{bmatrix} \quad (24)$$

$$E = \begin{bmatrix} 2k & 0 \\ 0 & 2kb^2 \end{bmatrix} \quad (25)$$

where b is equal tothe half value of c. The matrix (25) show that the torsional spring equivalent depends of the half length of the plate.

C. Elastic Center and Coordinate z at theAerodynamic Center

The position b/2represents the location of the aerodynamic center. In this case the torsion caused by the lift turns to zero. Then B and Cshorten into

$$B = \begin{bmatrix} \pi S & 0 \\ 0 & 0 \end{bmatrix} \quad (26)$$

Chord (m)	.5
Span (m)	.7
Total Stiffness K_{total}(N/m)	1000
Mass m (kg)	1
Mass Moment of Inertia J (kg-m^2)	0.1

Table I: Dimensions of Prototype and Base Values

$$C = \begin{bmatrix} 0 & \pi S \\ 0 & 0 \end{bmatrix} \quad (27)$$

Matrix (27) still maintains the coupling between the axial and torsional movement so the system can still be unstable depending on the final position of the center of gravity.

D. Coordinate zat the Center of Gravity

This state displays the condition of inertia uncoupling in (14), changing into

$$A = \begin{bmatrix} m & 0 \\ 0 & J_z \end{bmatrix} \quad (28)$$

If case C and D are combined, then only (27) is the remaining term where coupling is present.

IV. INSTABILITY ANALYSIS

Using Table I as the reference parameters, an analysis was done by changing the position of the elastic center and center of gravity. The parameters are the base for a future experiment on a wind tunnel with a cross section of 1m x 1m.K_{total} is the sum of both springs.The result is the instability speed, be it flutter or divergence. The range of search is from 0 m/s to 150 m/s and the location is extracted when the

damping ratio is <-0.1% or if the natural frequency is zero. In Fig. 4, exact zones can be stablished using the aerodynamic center lines and the diagonal line where the elastic center and center of gravity are in the same place. The upper left zone is failure by divergence while the down right zone is failure by flutter. A special curve is stablished where it crosses both zones and the zone between the curve and diagonal is a zero-airspeed unstable zone. The bright leftover zone is a stable zone where neither flutter nor divergence can occur. In Fig. 5, 6 and 7, starting from table I parameters, are shown the transformation of each of the zones if K_{total}, J and m are increased respectively.

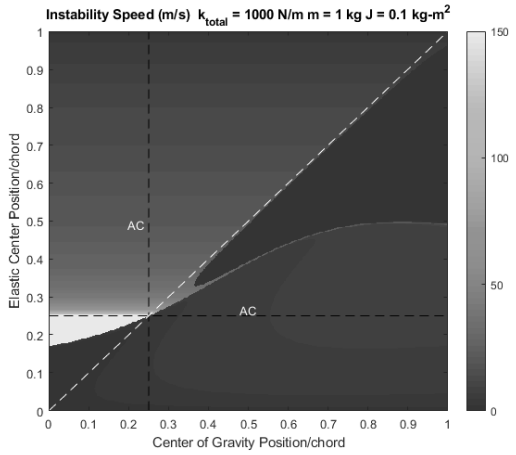


Figure 4: Instability Speed based on Table I

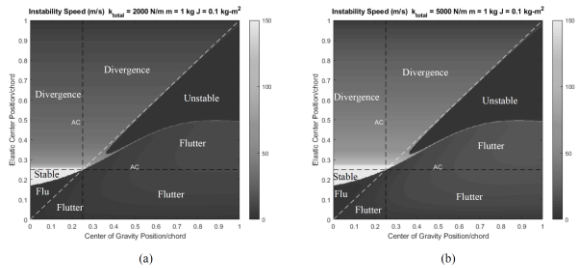


Figure 5: Instability Speed by increasing the total stiffnessK_{total}. (a) K_{total}=2000 N/m,(b) K_{total}=5000 N/m

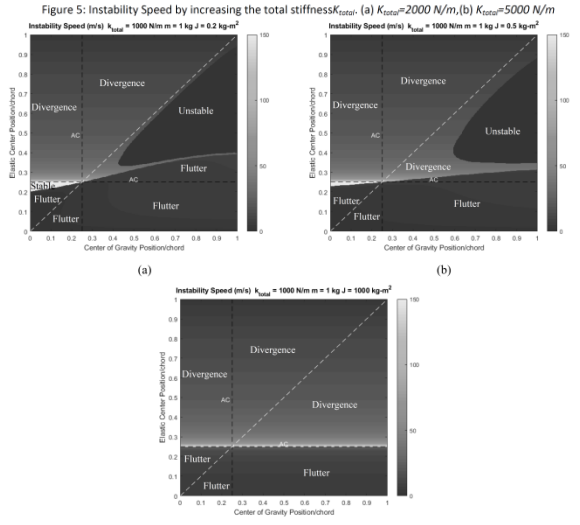


Figure 6: Instability Speed by increasing the moment of inertiaJ. (a) J=0.2 kg-m^2,(b) J=0.5 kg-m^2, (c) J=1000 kg-m^2

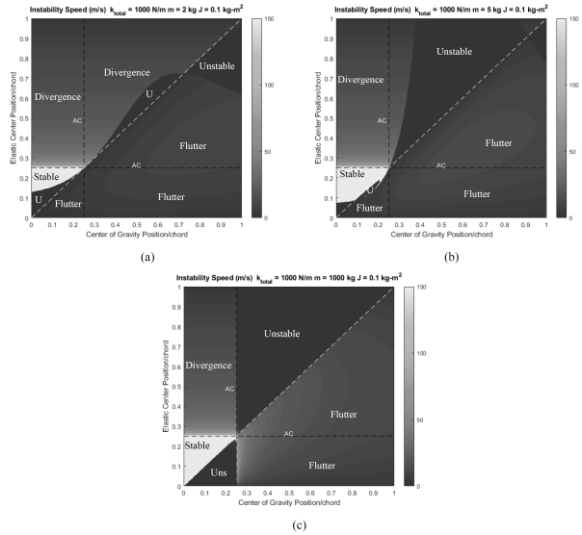


Figure 7: Instability Speed by increasing the massm. (a) m=2 kg,(b) m=5 kg, (c) m=1000 kg

V. DISCUSSION

From Fig. 5, it can be observed that the increase of stiffness does not modify the layout of the map. The only change is the increase of the instability speed. This demonstrates that by reducing the overall stiffness of the structure, there is more sensitivity to an aeroelastic failure. From Fig. 6, the increase of the mass moment of inertia causesthe curve, that separates the flutter zone and divergence zone, totake the form of the horizontal aerodynamic center line. Furthermore, this new line represents a new zone where the system is always stable, while the unstable zone disappeared. The stable zone now separates divergence, representing the top part, and flutter, the bottom part. Finally, the center of gravity does not influence the instability speed since no rotation can occur when J tend to infinity. By analyzing Fig. 7, when m goes to infinitytransforms the curve into the vertical aerodynamic center line. This case there is no translational movement and shows that the diagonal line and aerodynamic center lines exactly stablishes the flutter, divergence, unstable and stable zones. Overall, the influence of the curve, which depends of m and J, can generate new unstable zones which is influenced by the vertical line and the diagonal line. Moreover, the curve always passes through the point where the three lines cross. The results shown are based on quasi-steady assumptions. The model can be improved by adding the circulatory and non-circulatory terms where the reduced frequency is an important parameter included in the terms.

VI. CONCLUSIONS AND FUTURE WORK

With the new information shown, either with the matrices or the map of zones, it is more straightforward to stablish design parameters in order to control which zone to choose and develop further experiments. The analysis presented is part of an

ongoing research where flutter is the desired failure to generate in a prototype. Moreover, a comparison of the experimental results will be performedwith the analytical values.

REFERENCES

[1] Karpel, M. (1981). "Design for active and passive flutter suppression and gust alleviation" (Vol. 3482). National Aeronautics and Space Administration, Scientific and Technical Information Branch.

[2] Yang, Z. C., Zhao, L. C., & Jiang, J. S. (1995). "A semi-active flutter control scheme for a two-dimensional wing". Journal of sound and vibration, 184(1), 1-7.

[3] Verstraelen, E., Habib, G., Kerschen, G., & Dimitriadis, G. (2016). "Experimental passive flutter suppression using a linear tuned vibration absorber". AIAA Journal, 1707-1722.

[4] Verstraelen, E., Boutet, J., Grappasonni, C., Kerschen, G., & Dimitriadis, G. (2015, June). "Theoretical and experimental investigation of a structurally and aerodynamically nonlinear pitch and flap wing". In Proceedings of the International Forum on Aeroelasticity and Structural Dynamics, IFASD 2015 (pp. Paper-IFASD).

[5] Verstraelen, E. (2018). "Aeroelastic Limit Cycle Oscillations Mitigation Using Linear and Nonlinear Tuned Mass Dampers" (Doctoral dissertation, Université de Liège, Liège, Belgique).

[6] Lacarbonara, W., & Cetraro, M. (2011). Flutter control of a lifting surface via visco-hysteretic vibration absorbers. International Journal of Aeronautical and Space Sciences, 12(4), 331-345.

[7] Zedan, M. F., & Abu-Abdou, K. (1988). Improved thin-airfoil theory. Journal of aircraft, 25(12), 1122-1128.

[8] Pigolotti, L., Mannini, C., & Bartoli, G. (2017). Destabilizing damping effect on flat-plate vibrations due to flow-induced flutter. Procedia engineering, 199, 790-795.

[9] Wright, J. R., & Cooper, J. E. (2015). Introduction to aircraft aeroelasticity and loads. Chichester, West Sussex, United Kingdom: Wiley. pp 107-205

★★★