THE UNCERTAINTY-INVESTMENT RELATIONSHIP WITH ENDOGENOUS CAPACITY

SUDIPTO SARKAR

McMaster University, Canada

Abstract - This paper revisits the uncertainty-investment relationship with a real-option model, where the firm chooses both the timing and size of the investment, unlike existing studies. Using a new composite measure of investment that takes into account both timing and size, it is shown that investment is generally a non-monotonic (initially increasing and subsequently decreasing) function of uncertainty. Thus, uncertainty can have a positive or negative impact on investment. Comparative static analysis indicates that uncertainty is more likely to have a positive effect on investment when demand growth rate and demand volatility are low, and interest rate and operating cost are high. The effect also depends on the production technology; with decreasing-returns-to-scale technology it is more likely to be positive, and with increasing-returns-to-scale technology it is more likely to be negative. It is also shown that investment is more sensitive to uncertainty for decreasing-returns-to-scale technology, high cost of production capacity, low operating cost and less market power. While there is some empirical evidence in support of a non-monotonic uncertainty-investment relationship and the role of market power, the other implications of the model have not been empirically tested yet.

Keywords - Investment, Real-option model, Uncertainty, Volatility.

I. INTRODUCTION

Investment is a critical decision in Economics and Finance, and a significant determinant is uncertainty or volatility. Not surprisingly, there is a substantial literature, both theoretical and empirical, on the effect of uncertainty on investment. Our paper examines the uncertainty-investment relationship, but with a significant departure from the existing literature – the investment size or scale is not fixed but chosen by the firm. This has not been done as yet.

The main result is that investment is, in general, initially increasing and subsequently decreasing in uncertainty. It is shown that uncertainty is more likely to have a positive effect on investment when demand growth rate and demand volatility are low, discount rate and operating cost are high, and the technology is decreasing-returns-to-scale. Finally, we show that investment is more sensitive to uncertainty for decreasing-returns-to-scale technology, high cost of production capacity, low operating cost and less market power. Our model explains a number of recent empirical findings, e.g., Kellogg (2014), Stein and Stone (2013), Pennings and Altomonte (2006) and Serven (2003).

II. THE MODEL

We use a simple real-option model of investment to study the uncertainty-investment relationship, similar to Dixit and Pindyck (1994); however, investment size is endogenous, and chosen optimally by the firm.

A firm holds an option to invest in a project. The size of the investment is given by capacity Q, and investment cost is φQ^{α}, where α is the returns-to-scale parameter. Production cost is w per unit, and output price is given by:

$$p_t = y_t - \gamma q_t, \quad (1)$$

where q_t is the output level, γ is a non-negative constant, and y_t is an exogenous shock to demand. The demand shock y can be viewed as the (random) strength of demand for the product, and evolves as a lognormal process:

$$dy/y = \mu dt + \sigma dZ, \quad (2)$$

where μ and σ are the mean and standard deviation of the process, respectively, and Z is a standard Wiener process.

Cash flows are discounted at a rate r (where $r > \mu$) , the firm always operates at full capacity (q_t = Q). However, if y falls sufficiently, the firm can abandon operations.

B. Valuation of Project and Option to Invest

It can be shown that the project value is :

$$V(y) = \frac{Qy}{(r-\mu)} - \frac{(w+\gamma Q)Q}{r} + Ay^{\theta_2}, \quad (3)$$

where A is a constant to be determined by boundary conditions, and θ_1 and θ_2 are given by:

$$\theta_1 = 0.5 - \frac{\mu}{\sigma^2} + \sqrt{\left(0.5 - \mu/\sigma^2\right)^2 + \frac{2r}{\sigma^2}},$$

$$\theta_2 = 0.5 - \frac{\mu}{\sigma^2} - \sqrt{\left(0.5 - \mu/\sigma^2\right)^2 + \frac{2r}{\sigma^2}}$$

The constant A and the optimal abandonment trigger y_b are given by:

$$A = -\frac{Q(y_b)^{1-\theta_2}}{\theta_2(r-\mu)}$$, and

$$y_b = \frac{(w+\gamma Q)(1-\mu/r)}{(1-1/\theta_2)}.$$

The firm will invest when y rises to y_i, the investment trigger. Then it can be shown that the value of the option to invest is given by:

$$f(y) = Fy^{\theta_1}, \quad (4)$$

C. Optimal Investment Decision

The optimal investment decision with endogenous timing and size is given by Proposition 1 below.

Proposition 1

The optimal investment trigger y_i is is the solution to the equation:

$$\frac{Qy_i(1-1/\theta_1)}{(r-\mu)} - \frac{(w+\gamma Q)Q}{r} + A(1-\theta_2/\theta_1)(y_i)^{\theta_2} = \varphi Q^{\alpha} \quad (5)$$

and the optimal capacity Q is the solution to the equation:

$$\frac{y_i}{(1-\mu/r)} - (w+\gamma Q) + \left[Q\gamma + \frac{w+\gamma Q}{1-\theta_2}\right]\left(\frac{y_i}{y_b}\right)^{\theta_2} = r\varphi\alpha Q^{\alpha-1} \quad (6)$$

D. The Expected Present Value of Investment (EPVI)

The fact that delayed (earlier) investment is larger (smaller) makes it difficult to identify the effect on investment with a one-dimensional measure. We therefore use a composite measure that takes into account both dimensions – timing and size of investment. An appropriate measure would be the expected present value of investment or EPVIsince this incorporates both timing and size. The EPVI is the expected present value of φQ^{α} invested at the first passage time of y to y_i is given by $\varphi Q^{\alpha}(y/y_i)^{\theta_1}$ (Leland, 1994, footnote 16). However, this measure varies continuously, hence we use the measure $\varphi Q^{\alpha}/(y_i)^{\theta_1}$ instead, that is:

$$\text{EPVI} = \frac{\varphi Q^{\alpha}}{(y_i)^{\theta_1}} \quad (7)$$

III. RESULTS

A. Base-case Parameter Values

The following base-case parameter values are taken from Sarkar (2018): discount rate r = 5%, demand growth rate μ = 0%, and demand volatility σ = 15%. In addition, we use a price sensitivity of γ = 1, unit cost of capital φ = 1, variable cost w = 1, and returns-to-scale parameter α = 1.

B. Both Investment Trigger and Capacity Chosen Optimally

The optimal investment trigger and optimal capacity are determined by solving equations (5) and (6) simultaneously, and the resulting EPVI computed from equation (7). With the base-case parameter values, we get the following output with three different levels of volatility:

For σ = 10%, we get y_i = 2.0620, Q = 0.6256, and EPVI = 0.0430;

For σ = 15%, y_i = 3.5551, Q = 1.6030, and EPVI = 0.0544;

For σ = 20%, y_i = 11.3441, Q = 6.7771, and EPVI = 0.0359.

Figure 1 shows the results over a range of volatilities. We note that both y_i and Q are monotonically increasing functions of volatility. The former is a standard implication from option theory – that higher volatility delays investment. The latter relationship arises from the fact that higher volatility increases optimal capacity as well as the fact that delayed investment is larger in size. Since both y_i and Q are increasing in σ, the overall effect of a higher volatility is larger but delayed investment.

Also, both trigger and size rise at an increasing rate, reaching very high levels for large volatility. This happens because the two reinforce one another; a higher Q results in a higher y_i, which in turn results in a higher Q, and so on, until very high levels are reached. Thus, for high volatility levels, investment can be postponed indefinitely (see Dangl, 1999), resulting in very small EPVI. It is clear that high levels of uncertainty can be very unfavorable for corporate investment; not surprisingly, politicians are very reluctant to make dramatic policy changes once they achieve power, since such policy changes represent high volatility.

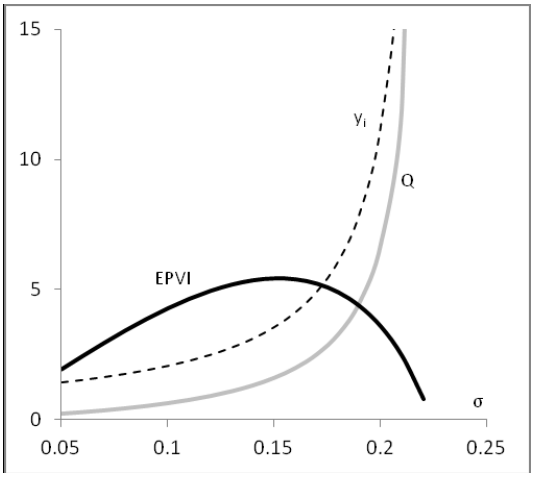


Figure 1. Shows optimal investment decision (yi and Q) and the resulting EPVI as functions of volatility σ (note that the graph shows 100*EPVI). The base-case parameter values are used: r = 5%, μ = 0%, γ = 1, φ = 1, w = 1 and α = 1.

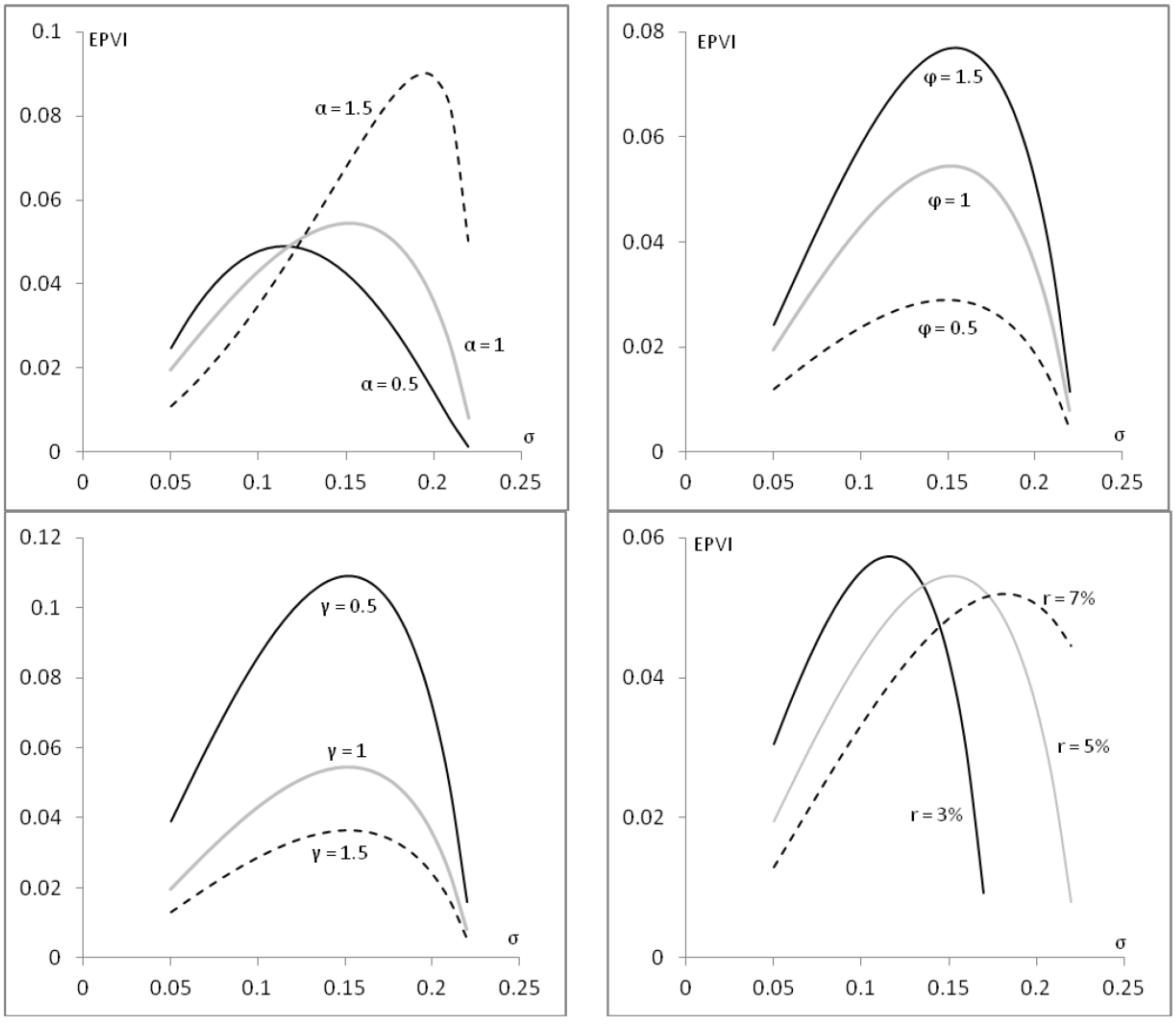
The overall effect on investment is found to be a non-monotonic (inverted-U shaped) function of volatility, with EPVI initially increasing and subsequently decreasing in volatility. Extremes in uncertainty do not help corporate investment, and the effect is particularly negative for very large volatility.

Result 1. When both the timing and size of investment are chosen optimally by the firm, the uncertainty-investment relationship is generally non-monotonic; investment (measured by EPVI) is initially increasing and subsequently decreasing in volatility.

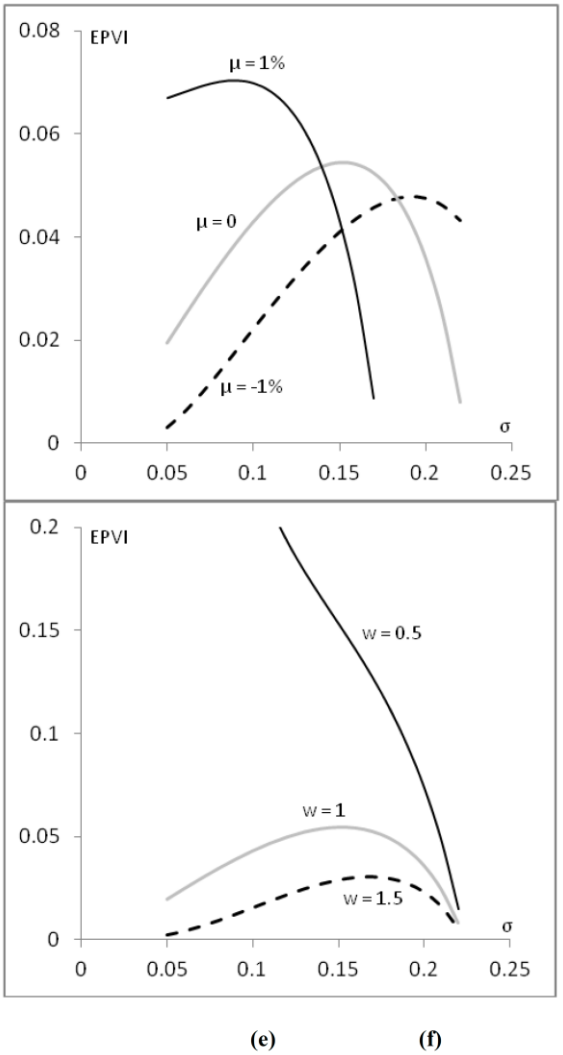
The relative importance of the rising and falling sections is important because it affects the frequency with which we observe positive or negative relationships. If the critical point is far to the right (i.e., high σ), we should more frequently observe an increasing relationship. In the next section, we repeat the numerical computations with a wide range of parameter values, to see (i) if the general shape is unchanged, and (ii) how the point at which the relationship turns from positive to negative varies with the different parameter values.

C. Comparative Static Results

Figure 2 displays the results with different parameter values. We note that in all cases the general shape is the same (that is, an inverted-U shape); thus the main point of Result 1 (that investment is generally initially increasing and subsequently decreasing in uncertainty) is quite robust. However, there is significant variation in the exact point at which the relationship goes from positive to negative, as described below.



(a) **(b)** **(c)** **(d)**



(e) (f)

Figure 2. Comparative static results. Shows the effect of volatility σ on investment (EPVI) for a range of parameter values. Base-case parameter values: r = 5%, μ = 0%, γ = 1, φ = 1, w = 1 and α = 1.

(a) Returns-to-scale parameter α: For higher α EPVI starts falling later, hence we are more likely to observe an increasing relationship when α is higher. Also, the effect of α on EPVI is ambiguous; for small (large) volatility, EPVI is decreasing (increasing) in α, which is very different from traditional real-option models, where a higher α has an unambiguous negative effect on investment. Finally, we also note that investment (EPVI) is more sensitive to volatility when α is high.

(b) Price-sensitivity or Market Power γ: The critical volatility does not change with γ, hence γ has no effect on the exact shape of the uncertainty-investment relationship. Also, EPVI is a decreasing function of γ, and EPVI is more (less) sensitive to volatility when γ is low (high), consistent with the empirical findings of Maioli (2005) and Ptanaik (2015).

(c) Cost of capacity φ: Unit investment cost has no impact on the shape of the uncertainty-investment relationship; also, EPVI is an increasing function of φ, and EPVI is more sensitive to volatility when φ is large.

(d) Interest rate r: For higher r, EPVI starts falling later, thus the uncertainty-investment relationship is more likely to be positive. Also, the effect of r on EPVI is ambiguous; for small σ EPVI is decreasing in r, and for large σ EPVI is increasing in r.

(e) Demand growth rate μ: For higher μ, EPVI starts falling earlier, hence the relationship is more likely to be a negative one. Therefore, a higher (lower) μ raises the likelihood of observing a negative (positive) uncertainty-investment relationship. The effect of μ on EPVI depends on the volatility; for small (large) σ, EPVI is increasing (decreasing) in μ.

(f) Operating cost w: For higher w, EPVI starts falling later, thus the observed uncertainty-investment relationship is more likely to be positive. Also, EPVI is a decreasing function of w, and EPVI is more sensitive to volatility when w is small.

Result 2.

(a) The effect of uncertainty on investment is more likely to be positive for decreasing-returns-to-scale technology, for low demand growth rate and volatility, and for high interest rate and operating cost.

(b) Investment is more sensitive to volatility for decreasing-returns-to-scale and low-operating-cost technology, for costly capacity, and for competitive industries.

Result 2 specifies the conditions under which the relationship is likely to be positive, and the conditions under which investment will be more sensitive to uncertainty, which is of considerable interest because uncertainty is substantially impacted by government policies. If government policies are unstable, economic uncertainty increases. Thus, for costly capacity, decreasing returns to scale, low operating cost (or high margin) and more competitive industries, government policies are more relevant because investment is more sensitive to uncertainty. In these cases, therefore, government decisions and policies have larger impact on corporate investment, and have to be set more carefully.

IV. SUMMARY AND CONCLUSIONS

This paper uses a real-option model to study how uncertainty affects corporate investment, when both timing and size of investment are determined by the firm (as is often the case in practice). With endogenous timing and size, the effect on investment needs to capture both dimensions, thus a composite measure of investment is necessary. We propose such

a measure, the expected present value of investment or EPVI (in dollars).

We show that investment is, in general, initially increasing and subsequently decreasing in uncertainty; thus, uncertainty can have a positive or negative effect on investment. However, the point at which it switches from positive to negative can vary widely with input parameter values. We are more likely to observe a positive relationship between uncertainty and investment for decreasing-returns-to-scale technology, low demand growth rate and volatility, or high interest rate and operating cost. In these situations, therefore, greater uncertainty can be good for investment.

We also show that the effect of certain parameters on investment can differ greatly from the traditional fixed-capacity case; for instance, investment is a decreasing function of interest rate when uncertainty is low but an increasing function when uncertainty is high. Finally, we show that investment is more sensitive to volatility for decreasing-returns-to-scale technology, competitive markets, costly capital and high operating margins.

As in most models, we make simplifying assumptions for reasons of tractability. For instance, we consider only one investment opportunity that requires a lumpy investment, with no option of future expansion. Thus the model will be useful for lumpy, one-shot investments, but not for investments with expansion opportunities or for a company considering multiple investment opportunities at the same time. Our model could be extended to analyze these scenarios.

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