ANALYSIS OF THERMAL DAMAGE IN TISSUE WITH NON-EQUILIBRIUM EFFECT

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Abstract- The present paper analyzes thermal damage in a pulse laser-irradiated tissue based on the generalized dual-phase-lag model. The discrepancy of the generalized dual-phase-lag model with the Pennes bio-heat transfer model is explored. Porosity and coupling factor are particularly important to the generalized DPL model, so their effects are investigated. The paper notices that the phase lag times depend on the porosity, heat capacities of blood and tissues, coupling factor, and the ratio of thermal conductivity of tissue and blood.

Index Terms- Dual-phase-lag, laser irradiation, bioheat transfer.

I. INTRODUCTION

The whole anatomical structure of biological tissue is able to be splited into vascular regionand extravascular region, so the biological tissue can be treated the as a fluid saturated porous medium [1, 2]. The blood temperature differs from the tissue temperature, and the blood temperature varies due to convective heat transfer between the blood and tissue and blood perfusion. Therefore, to consider that the heat transfer in living biological tissue should be the non-equilibrium heat transfer is much more realistic than equilibrium heat transfer assumptions [3]. The two-temperature models based on volume average to the local instantaneous governing equation for blood and tissue were proposed [1, 2]. Roetzel and Xuan [3] further proposed a more refined model that splits the blood flow into arterial and venous components. Based on a two-temperature model, Zhang [4] developed the generalized dual-phase-lag bioheat transfer equation, which its phase lag times are not independent properties and can be calculated with the properties of blood and tissue, interphase convective heat transfer coefficient and blood perfusion rate.

The generalized dual-phase-lag bioheat transfer equation has been applied to explore thermal response in a laser-irradiated tissue [5]. Narasimhan and Sadasivam [6] also modeled the thermal behavior of the human eye during retinal laser irradiation with it. The present work would use it to investigate the thermal response in a living tissue that laser irradiation is highly absorbed. Therefore, the generalized dual-phase-lag bioheat transfer equation with the pulse heat flux boundary condition is solved. And then, the thermal damage is predicted with the Arrenius equation, which was accepted and used to evaluate the damage parameter [7].

There are mathematical difficulties for solving such a problem [8]. The hybrid numerical scheme [8] based on Laplace transform is employed to solve the problem. For convenience of analysis, the literature [5, 6] regard the phase lag times as the independent parameters. In fact, the phase lag times depend on the porosity, heat capacities of blood and tissues, coupling factor, and the ratio of thermal conductivity of tissue and blood. This character has been taken into account in the present work.

II. PROBLEM FORMULATION

Based on equilibrium heat transfer assumption, the classical dual-phase-lag model of bioheat transfer is written as:

$$(1+\tau_T \frac{\partial}{\partial t})k\nabla^2 T = (1+\tau_q \frac{\partial}{\partial t})[\rho c \frac{\partial T}{\partial t} - w_b \rho_b c_b (T_b - T) - q_m - q_r] \quad (1)$$

where k, ρ, c, and T denote the conductivity, density, specific heat, and temperature of tissue. t is the time. c_b and w_b are, respectively, the specific heat and perfusion rate of blood. q_m is the metabolic heat generation and q_r is the heat source for spatial heating. T_b is the arterial temperature. τ_q means the phase lag of the heat flux and τ_T means the phase lag of the temperature gradient.

However, Nakayama and Kuwahara [1] recognized that the temperatures of blood and tissue are different and the equilibrium heat transfer assumption is invalid for heat transfer in living biological tissue and developed a two-temperature model for bioheat transfer and blood flow as

$$\varepsilon \rho_b c_b \left[\frac{\partial T_b}{\partial t} + V \cdot \nabla T_b \right] = \varepsilon k_b \nabla^2 T_b + G(T - T_b) + \varepsilon q_r \quad (2)$$

$$(1-\varepsilon)\rho c \frac{\partial T}{\partial t} = (1-\varepsilon)k\nabla^2 T + G(T_b - T) + (1-\varepsilon)q_m + (1-\varepsilon)q_r \quad (3)$$

where $G = a_b h_b + w_b c_b$ is the coupling factor between blood and tissue. h_b and a_b are the convective heat transfer coefficient and the specific area of the blood

vessel in the tissue, respectively. ε is a proportional rate and subscript b is referred to blood.

The generalized dual-phase-lag bioheat transfer equation with tissue temperature as sole unknown was derived by Zhang [4] based on the two-temperature model and can be written as

$$(1+\tau_q\frac{\partial}{\partial t})(\rho c)_{eff}\frac{\partial T}{\partial t}=(1+\tau_T\frac{\partial}{\partial t})k_{eff}\nabla^2 T+G(T_b-T)+(1+\frac{\varepsilon\rho_b c_b}{G}\frac{\partial}{\partial t})[(1-\varepsilon)q_m+q_r] \quad (4)$$

where

$$(\rho c)_{eff}=\varepsilon\rho_b c_b+(1-\varepsilon)\rho c \quad (5)$$

$$k_{eff}=\varepsilon k_b+(1-\varepsilon)k \quad (6)$$

and

$$\alpha_{eff}=\frac{k_{eff}}{(\rho c)_{eff}} \quad (7)$$

are effective heat capacity, thermal conductivity, and thermal diffusivity, respectively. This equation includes the effects from the blood flow, thermal diffusion and the local thermal non-equilibrium between the blood and the peripheral tissue. The phase lag times depends on the porosity, heat capacities of blood and tissues, coupling factor, and the ratio of thermal conductivities of tissue and blood. The phase lag times τ_q and τ_T can be obtained with

$$\tau_q=\frac{\varepsilon(1-\varepsilon)}{[\varepsilon/C_{sb}+(1-\varepsilon)]}\frac{\rho_b c_b}{G} \quad (8)$$

$$\tau_T=\frac{\varepsilon(1-\varepsilon)}{[\varepsilon/K_{sb}+(1-\varepsilon)]}\frac{\rho_b c_b}{G} \quad (9)$$

where $C_{sb}=\rho c/(\rho_b c_b)$ and $K_{sb}=k/k_b$.

Consider a broad laser beam with a uniform irradiance (ϕ_{in}) is applied normally to a finite slab of biological tissue with a thickness of L at time $t = 0^+$. When the spot size of the broad laser beam is much larger than the thickness of the thermally affected zone for the time period of interest, a 1-D model would be sufficient for analyzing the thermal response of the heated medium [5, 9].

The 1-D form of (4) with constant thermal parameters is written as

$$(1+\tau_q\frac{\partial}{\partial t})(\rho c)_{eff}\frac{\partial T}{\partial t}=(1+\tau_T\frac{\partial}{\partial t})k_{eff}\frac{\partial^2 T}{\partial x^2}+G(T_b-T)+(1+\frac{\varepsilon\rho_b c_b}{G}\frac{\partial}{\partial t})[(1-\varepsilon)q_m+q_r] \quad (10)$$

The studied problem has the initial conditions

$$T(x,0)=T_b \text{ and } \frac{\partial T(x,0)}{\partial t}=0 \quad (11)$$

When laser irradiation is highly absorbed in the tissue as for some UV and IR wavelengths, scattering may be neglected. For convenience of comparison, this work regards the transport behavior of laser light in the tissue as highly absorbed. This study assumes the laser irradiance ϕ_{in} is deposited as

$$\phi_{in}=q_0[u(t)-u(t-\Delta t)] \quad (12)$$

where u(t) is the unit step function, ϕ_{in} is the incident laser irradiance, q_0 the laser intensity, and Δt the laser exposure time.

For that the laser light can be absorbed within a very small depth from the irradiated boundary surface, laser heating can be reasonably considered as a boundary surface heat flux. Under such conditions, the terms involved q_r in (10) are removed, and the heat flux boundary conditions are given as [5]

$q = \phi_{in}(1-R_d)$ at x = 0 for t >0 (13)

$q = 0$ at x = L for t >0 (14)

where R_d is the diffuse reflectance of light at the irradiated surface.

For estimation of thermal damage in a living tissue, the damage parameter Ω was defined as

$$\Omega=\ln(\frac{C_0}{C_0-C_d}) \quad (15)$$

where C_0 is the protein concentration in the normal tissue, C_d the concentration of denaturated protein. Welch [11] regarded the protein denaturation process as a chemical reaction. The Arrenius equation was used to evaluate the damage parameter Ω:

$$\Omega=A\int_{t_1}^{t_f}\exp(-\frac{E}{RT})dt \quad (16)$$

where A is the frequency factor, E is the energy of activation of denaturation reaction, R is the universal gas constant, 8.314 J/(mol K), T is the absolute temperature of the tissue at the point where Ω is calculated, t_1 is the time at the onset of laser exposure, and t_f is the time when the thermal damage is evaluated [5].

When the protein denaturation reaches 63% of the molecules, which corresponds to Ω = 1.0, the tissue is assumed to be irreversible damaged. The values of the frequency factor and the activation energy are given as: $A = 3.1\times10^{98}$ s−1; $E = 6.28\times10^5$ J/mol [12].

III. ANALYTICAL METHOD

First, an elevation variable H is defined as $H=T-T_b$. Then, (10) is rewritten as

$$(1+\tau_T\frac{\partial}{\partial t})k_{eff}\frac{\partial^2 H}{\partial x^2}=(1+\tau_q\frac{\partial}{\partial t})(\rho c)_{eff}\frac{\partial H}{\partial t}-GH-(1-\varepsilon)q_m \quad (17)$$

The initial conditions become

$$H(x,0)=0 \text{ and } \frac{\partial H(x,0)}{\partial t}=0 \quad (18)$$

The Laplace transform technique is used to map the transient problem into the steady one. The differential equation (19) are transformed based on the initial conditions (18) into

$$\frac{d^2\tilde{H}}{dx^2}-\lambda^2\tilde{H}=-f \quad (19)$$

$\tilde{H}$ is the Laplace transform of the variable H. The relevant parameters are defined as

$$\lambda^2 = \frac{G/(\rho c)_{eff} + (1+\tau_q s)s}{\alpha_{eff}(1+\tau_T s)} \quad (20)$$

$$f = \frac{(1-\varepsilon)q_m}{k_{eff}(1+\tau_T s)s} \quad (21)$$

The boundary conditions (13) and (14) in the Laplace transform domain can be written as

$$\frac{d\tilde{H}}{dx} = -k\frac{1+\tau_T s}{1+\tau_q s}\tilde{\phi}_{in}(1-R_d) \text{ at x = 0} \quad (22)$$

$$\frac{d\tilde{H}}{dx} = 0 \text{ at x = L} \quad (23)$$

The present work divides the whole space domain into several sub-space domains. For continuities of heat flux and temperature within the whole space domain, the following conditions are required at the interface of the sub-space domain j-1, $[x_{i-1}, x_i]$, and the sub-space domain j, $[x_i, x_{i+1}]$.

$$\tilde{H}_{j-1}(x_i) = \tilde{H}_j(x_i) \text{ i = 1,2,….,n ; j = i} \quad (24)$$

$$\frac{d\tilde{H}_{j-1}(x_i)}{dx} = \frac{d\tilde{H}_j(x_i)}{dx} \text{ i = 1,2,….,n ; j = i} \quad (25)$$

where the subscript i is the number of node. n is the total number of nodes.

In order to perform the derivation of the governing algebraic equations, $\tilde{H}$ is approximated by using the nodal temperatures and shape function within a small sub-space domain. The shape function in each sub-space domain is derived from the governing equation (19) with the following procedures.

For the sub-space domain j, $[x_i, x_{i+1}]$, the analytical solution of the governing equation (20) subjected to the boundary conditions

$$\tilde{H}_j(x_i) = \tilde{H}_i \quad \text{and} \quad \tilde{H}_j(x_{i+1}) = \tilde{H}_{i+1} \quad (26)$$

are easily obtained and can be written as

$$\tilde{H}_j = \frac{1}{\sinh \lambda\ell}\left\{\begin{array}{l}(\tilde{H}_i - \frac{f}{\lambda^2}x_i)\sinh\lambda(x_{i+1}-x) \\ +(\tilde{H}_{i+1} - \frac{f}{\lambda^2}x_{i+1})\sinh\lambda(x-x_i)\end{array}\right\} + \frac{f}{\lambda^2} \quad (27)$$

where ℓ denotes the length of sub-space domain or the distance between two neighboring nodes.

Substituting (24) and the shape function (27) into (25) and then evaluating the resulting derivative can lead to the discretized form for the interior nodes as following

$$\tilde{H}_{i-1} - 2\cosh(\lambda\ell)\tilde{H}_i + \tilde{H}_{i+1} = \frac{f}{\lambda^2}[2-2\cosh(\lambda\ell)] \quad (28)$$

Equation (28) in conjunction with the discretized forms of the boundary conditions can be rearranged as the following matrix equation

$$[B]\{\tilde{H}\} = \{F\} \quad (29)$$

where $[B]$ is a matrix with complex numbers, $\{\tilde{H}\}$ is a column vector in the Laplace transform domain, and $\{F\}$ is a column vector representing the forcing term. Thereafter, the value of H in the physical domain can be determined with the application of the Gaussian elimination algorithm and the numerical inversion of the Laplace transform [10].

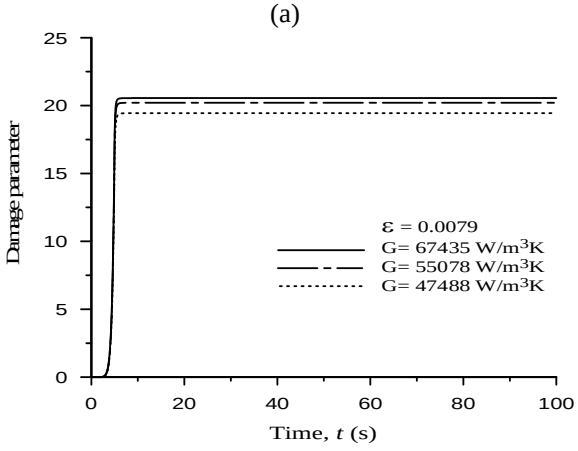
IV. RESULTS AND DISCUSSION

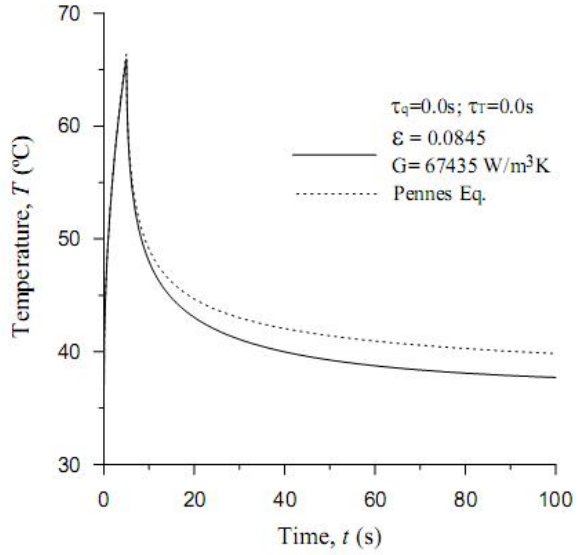
The thermophysical properties [5], used in the present calculations, of tissue are: k = 0.628 W/m·K, $\rho = 1000$ kg/m^3, and c = 4187 J/(kg·K). The thermophysical properties and temperature of blood are: $\rho_b = 1.06\times10^3$ kg/m^3, c_b = 3860 J/(kg·K), $w_b = 1.87\times10^{-3}$m^3/(m^3·s), T_b = 37 °C. The metabolic heat generation is $q_m = 1.19\times10^3$ W/m^3. The values of R_d, L and Δt are 0.05, 5 cm and 5 s, respectively. The laser intensity is q_0 = 2 W/cm^2.

It is well known that the classical DPL bio-heat transfer equation can reduce to the Pennes bio-heat transfer equation for $\tau_q = \tau_T = 0$[8]. In order to know Therefore, this work analyzes the case of $\tau_q = \tau_T = 0$ s and compares the results with that based on the Pennes equation, as presented in Fig. 1. The curve for $\tau_q = \tau_T =$ 0s deviate from the curve obtained with the Pennes equation, excluding at the early times. Obviously, the generalized DPL bio-heat transfer equation can not reduce to the Pennes bio-heat transfer equation.

Equations (8) and (9) show that the phase lag times τ_T and τ_q depend on the porosity, heat capacities of blood and tissues, coupling factor, and the ratio of thermal conductivity of tissue and blood. As seen from Fig. 3, the temperatures at the irradiated surface x = 0 have an obvious deviation with the different values of ε. Figs. 2 and 3 also depict the prediction of thermal damage corresponded to the temperature evaluations. The value of damage parameter dramatically rises at the end of heating.

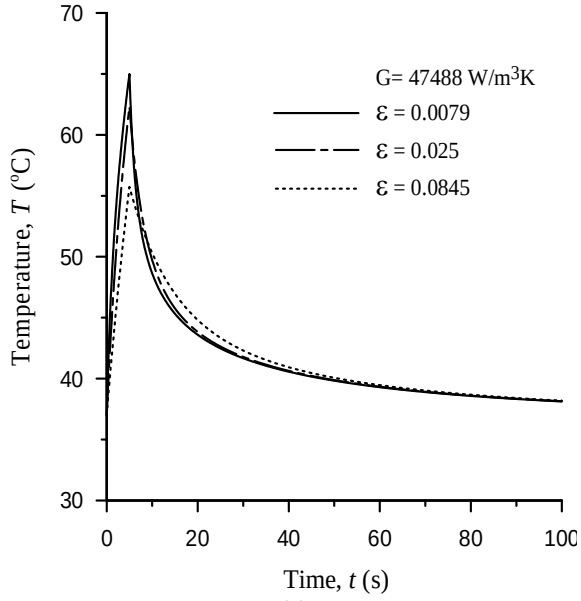
Fig. 1 Temperature evolution at the irradiated surface x = 0.



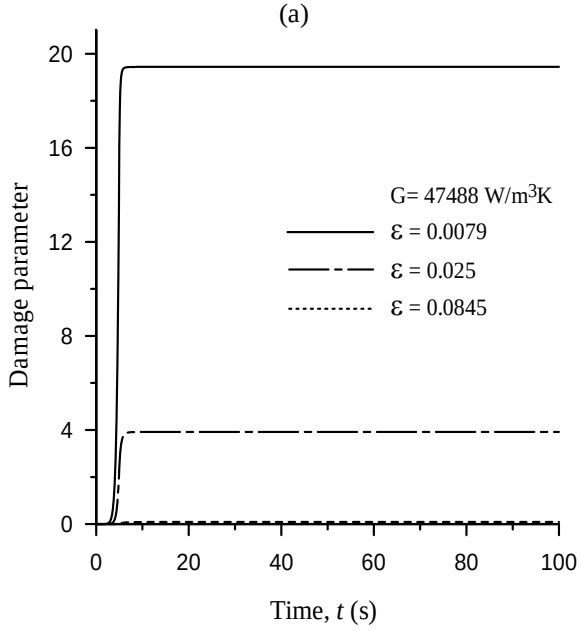


(b)

Fig. 2 Effect of coupling factor G on the results.



(a)



(b)

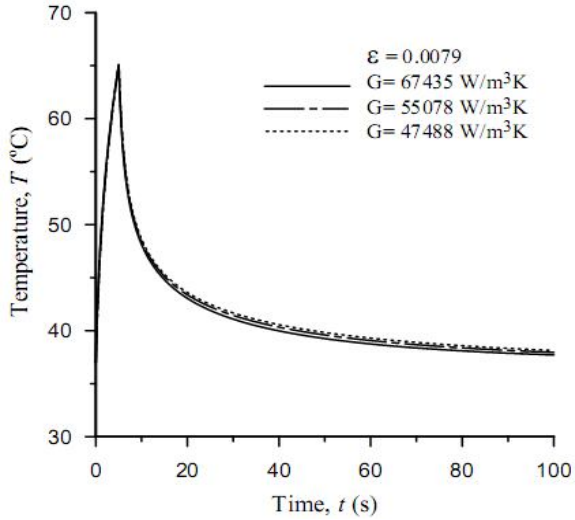


Fig. 3 Effect of porosity ε on the results.

CONCLUSION

A hybrid numerical scheme based on Laplace transform has been employed to numerically solve the generalized DPL bio-heat transfer equation with a pulse boundary heat flux. The results show the generalized DPL model of bioheat transfer can not reduce to the Pennes bio-heat equation for $\tau_q = \tau_T$, even $\tau_q = \tau_T = 0$. While the coupling factor between blood and tissue G and porosity ε dominate the values of phase lag times, τ_q and τ_T, they significantly affect the temperature variation in tissue.

REFERENCES

[1] A. Nakayama, F. Kuwahara, "A General Bioheat Transfer Model Based on the Theory of Porous Media," International Journal of Heat and Mass Transfer, Vol. 51, pp. 3190–3199, 2008.

[2] Y. Xuan, W. Roetzel, "Bioheat Transfer Equation of Human Thermal System," Chem. Eng. Technol. Vol. 20, pp. 268–276, 1997.

[3] Roetzel, W., and Xuan, Y., "Transient Response of the Human Limb to an External Stimulus," International Journal of Heat and Mass Transfer, Vol. 41, pp. 229–239, 1998.

[4] Y. Zhang, "Generalized Dual-phase Lag Bioheat Transfer Equations based on Nonequilibrium Heat Transfer in Living Biological Ttissues," International Journal of Heat and Mass Transfer, Vol. 52, pp. 4829-4834, 2009.

[5] N. Afrin, J. Zhou, Y. Zhang, D.Y. Tzou, and J.K. Chen, "Numerical Simulation of Thermal Damage to Living Biological Tissues Induced by Laser Irradiation Based on a Generalized Dual Phase Lag Model," Numerical Heat Transfer, Part A: Applications, Vol. 61, pp. 483-501, 2012.

[6] A. Narasimhan, S. Sadasivam, "Non-Fourier Bioheat Transfer Modelling of Thermal Damage During Retinal Laser Irradiation," International Journal of Heat and Mass Transfer, Vol. 60, pp. 591–597, 2013.

[7] S.H. Diaz, J.S. Nelson, and B.J.F. Wong, "Rate Process Analysis of Thermal Damage in Cartilage," Physics in Medicine and Biology, Vol. 48, pp. 19-29, 2003.

[8] K.C. Liu, J.C. Wang, "Analysis of Thermal Damage to Laser Irradiated Tissue Based on the Dual-Phase-Lag Model," Int. J. Heat and Mass Transfer, Vol. 70, pp. 621–628, 2014.

[9] A. Kanezaki, A. Hirata, S. Watanabe, and H. Shirai, "Parameter Variation Effects on Temperature Elevation in a Steady-State, One-Dimensional Thermal Model for Millimeter Wave

Exposure of One- and Three-Layer Human Tissue," Phys. Med. Biol., Vol. 55, pp. 4647–4659, 2010.

[10] G. Honig, U. Hirdes, "A Method for the Numerical Inversion of Laplace Transforms," J. Comp. Appl. Math., Vol. 10, pp. 113-132, 1984.

[11] A.J. Welch, "The Thermal Response of Laser Irradiated Tissue," IEEE Journal of Quantum Electronics, Vol. 20, pp. 1471-1481, 1984.

[12] C.M. Gardner, S.L. Jacques, and A.J. Welch, "Light Transport in Tissue: Accurate, Heuristic Expressions for one dimensional fluence rate and escape function Based upon Monte Carlo Simulations," Lasers in Surgery and Medicine, Vol. 18, pp. 129-138, 1996.

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