SWING UP AND TRANSITION TO OPTIMAL CONTROL FOR A SINGLE INVERTED PENDULUM

[1]MASOUD RAZBAN, [2]AMIR HOOSHIAR, [3]NAGHMEH M.BANDARI, [4]CHUN YI SU, [5]JAVAD DARGAHI

[1,2,3,4,5]Department of Mechanical and Industrial Engineering, Concordia University, Montreal, QC, CA.
E-mail: [1]m_razban@encs.concordia.ca, [2]s_hooshi@encs.concordia.ca, [3]nag_moha@encs.concordia.ca, [4]chun-yi.su@concordia.ca, [5]dargahi@concordia.ca

Abstract- Parameter identification and swing up control of a single inverted pendulum is presented. Using Lagrangian method, the mechanical model of system was derived and the parameters were identified via energetic description of system. Energy based strategy for swing up controller and LQR approach for stabilization were implemented. A smooth weighted transition was proposed to hinder overshoot caused by switching from swinging up to stabilization controllers. The model of system was developed and its results were verified by experiments. The proposed weighted transition method was verified to decrease jerk and overshoot in the system.

Key words- Optimal Control, Swing up, LQR, Parameter Identification, Inverted Pendulum

I. INTRODUCTION

Inverted pendula form a highly non-linear class of problems in classical control theory. The interest in inverted pendula problems is raised due to the intrinsic similarities of these systems with a great variety of practical systems, e.g. balancing a stick on the hand palm, mimicking the human gait, balancing the rocket launcher, vertical motion of the human arm [1]. Since the number of actuators in the inverted pendula systems are typically less than the degrees of freedom (DOF), such systems are categorized as under-actuated systems and are extremely unstable yet controllable. As the DOFs of the system surpass the number of actuation, the complexity and nonlinearity of the system increases and the controllability of the system worsens.

In this study, a system of single inverted pendula on a cart was controlled. The objective was to swing up pendulum and transition to an optimal control which should be relatively robust facing environmental spontaneous disturbances. Thus, these 5 steps of were taken:

1. Modelling the problem using energy approach.
2. Identify the value of different system parameters extracted from modeling part.
3. Simulate the problem computationally in MATLAB® SIMULINK® and predict the behavior of the system under control.
4. Implement the controller on the system and perform data acquisition while controller in action.
5. Compare the performance of the system with the simulated predictions and optimize the parameters and/or controller design to minimize the error between the prediction and simulation.

The single and double inverted pendulum on a cart has already been studied by many researchers. Different control approaches has already been proposed and implemented to such systems, e.g. PID[2-4], LQR[4-6], LQG[7, 8], neural network[9, 10], fuzzy logic[2, 10, 11].

In another effort Jadlovska S. et al [1], have designed and implemented a complete MATLAB® toolbox, "Inverted Pendula Modeling and Control", which by having the physical properties of the system, e.g. length, mass and center of mass (COM) location of the cart and pendula, could reveal the governing state space equations of the system.

Studies have shown high sensitivity of the control performance on model parameters [12, 13], thus parameter identification process should be performed using a technique less prone to data acquisition noise and uncertainty. Through the literature concerning model-based parameter identification, two distinct methods are identifiable, i.e. parameter identification using the equation of motion of the system, denoted as EOM hereinafter [14], and parameter identification energetic description of the system, ED.

II. MECHANICAL MODELING

The mechanical model of the system was derived based on the geometrical assumptions as depicted in fig. 1. The energetic description of the system, i.e. kinetic and potential energies of the components, were formulated and then used to derive Lagrangian description of motion.

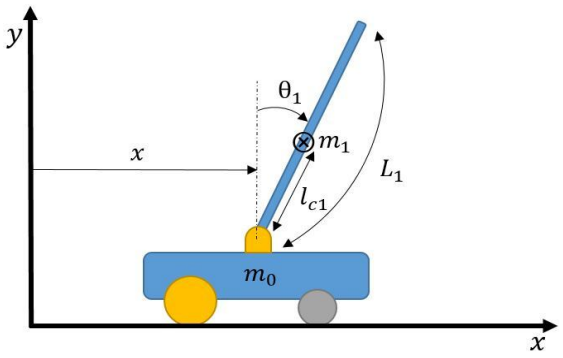


Fig 1: The geometrical model of the single inverted pendulum on a cart system.

The Lagrangian description of motion, implies that:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)-\frac{\partial L}{\partial q}=Q \quad (1)$$

Where, q is the vector of degrees of freedom, i.e. generalized coordinates, L is the Lagrangian functional and Q is the external forces applied to the system. In this problem q is:

$$q=\begin{pmatrix} x \\ \theta_1 \end{pmatrix} \quad (2)$$

And Q is:

$$Q=\begin{pmatrix} f_m-f_f \\ -\tau_{f1} \end{pmatrix} \quad (3)$$

Where, f_m is the horizontal driving force applied by the motor to the system, f_f is the viscous friction force against the motion of the cart, τ_{f1} is the viscous friction in the pivot joint. Jadlovska et al. [1] have shown that the force thrusted by a DC motor can be modeled as a linear function of input voltage. Moreover, the viscous friction of cart motion on the rack and pivot joints can be assumed as a linear function of the cart and joint velocities. Thus:

$$Q=\begin{pmatrix} K_m V-\delta_0 \dot{x} \\ -\delta_1 \dot{\theta}_1 \end{pmatrix} \quad (4)$$

The Lagrangian functional is defines as the difference between the kinetic and the potential energy of the whole system. So:

$$L=\sum T_i-\sum V_i=T_0+T_1-(V_0+V_1) \quad (5)$$

$$\sum V_i=V_0+V_1 \quad (6)$$

$$V_0=0 \quad (7)$$

$$V_1=m_1 g y_1=m_1 g l_{c1}\cos\theta_1 \quad (8)$$

$$\sum T_i=T_0+T_1 \quad (9)$$

$$T_0=\frac{1}{2}m_0\dot{x}^2 \quad (10)$$

$$T_1=\frac{1}{2}m_1(\dot{x}_1^2+\dot{y}_1^2)= \frac{1}{2}m_1\left[\left(\dot{x}^2+l_{c1}\dot{\theta}_1^2\cos\theta_1\right)^2+\left(l_{c1}\dot{\theta}_1^2\sin\theta_1\right)^2\right]+ \frac{1}{2}I_1\dot{\theta}_1^2=\frac{1}{2}m_1\dot{x}^2+\frac{1}{2}\left(m_1 {l_{c1}}^2+I_1\right)\dot{\theta}_1^2+ m_1 l_{c1}\dot{x}\ \dot{\theta}_1\cos\theta_1 \quad (11)$$

Thus, the equation of motion is derived as below:

$$(m_0+m_1)\ddot{x}+m_1 l_{c1}\ddot{\theta}_1\cos\theta_1-m_1 l_{c1}\dot{\theta}_1^2\sin\theta_1=K_m V-\delta_0\dot{x} \quad (12)$$

$$\left(m_1 {l_{c1}}^2+I_1\right)\ddot{\theta}_1+m_1 l_{c1}\ddot{x}\cos\theta_1-m_1 l_{c1} g\sin\theta_1)=-\delta_2\dot{\theta}_2 \quad (13)$$

Re-arranging the equations of motion in matrix notation, it can be identified as a second order mechanical system. Thus the system of differential equations can be expressed:

$$\boldsymbol{D}(q)\ddot{q}+\boldsymbol{C}(q,\dot{q})\dot{q}+\boldsymbol{G}(q)=\boldsymbol{H}u \quad (14)$$

$$u=K_m V_0 \quad \& \quad H=\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad (15)$$

Where, u is the input vector and H is the input gain matrix.

III. PARAMETER IDENTIFICATION

For Parameter Identification the energetic description of the system were employed. the concept is to form a set of equations stating the mechanical energetic balance of the system throughout the motion caused by excitation, τ. Since the system is starting from the resting position the kinetic energy is balanced at zero while, if no physical barrier is present, the potential energy is at its local minimum (static balance). Expedition of the excitation, usually motion of the cart, would result in cart and pendula motion while the friction work, between cart wheel(s) and rail(s) and pivot joints of the pendula, exhausts a portion of the external work done on the system. Equations 16 represents the energy balance of the cart-pendula system.

$$\int_{t_1}^{t_2}\tau\dot{x}dt-\int_{t_1}^{t_2}f\dot{x}dt=\sum_i T_i^{t_2}+\sum_i V_i^{t_2}-\sum_i T_i^{t_1}-\sum_i T_i^{t_1} \quad (16)$$

where, τ is the external force excitation, f represents the viscous frictional forces vector acting on system, T_i is the kinetic energy of the i_{th} component, Vi is the potential energy of the i_{th} component while t_1 and t_2 indicate the initial and final time of the excitation [14].

Energetic description of the single pendulum system for parameter identification introduced in equations 17 and 18.

$$E=\sum T_i+\sum V_i=T_0+T_1+V_0+V_1 \quad (17)$$

$$E=\frac{1}{2}(m_0+m_1)\dot{x}_0^2+\frac{1}{2}\left(m_1 {l_{c1}}^2+I_1\right)\dot{\theta}_1^2+m_1 l_{c1}\left(\dot{x}_0\dot{\theta}_1\cos\theta_1+g\cos\theta_1\right) \quad (18)$$

Thus, the parameters of the system, d_1-d_5 were selected:

$$d_1=m_0+m_1 \quad (19)$$

$$d_2=m_{1g}l_{c1} \quad (20)$$

$$d_3=m_1 {l_{c1}}^2+I_1 \quad (21)$$

$$d_4=\delta_0 \quad (22)$$

$$d_5=\delta_1 \quad (23)$$

By considering E_{t_2} and E_{t_1} as the state of energy in time t_2 and t_1, this relationship can be expressed:

$$E_{t_2}-E_{t_1}=w_m-w_f \quad (24)$$

Where w_m, is the work of external forces which is applied by motor and the w_f is the work of frictional forces from time t_1 to t_2.

$$w_m=\int_{t_1}^{t_2}f_m\dot{x}\ dt=\int_{t_1}^{t_2}K_m V_0\dot{x}\ dt \quad (25)$$

$$w_f=w_{f_0}+w_{f_1}=\int_{t_1}^{t_2}\delta_0\dot{x}_0^2\,dt+\int_{t_1}^{t_2}\delta_1\dot{\theta}_1^2\,dt \quad (26)$$

Thus, the energy relationship between two time steps was determined as below:

$$E_{t_1}-E_{t_0}+\int_{t_1}^{t_2}d_4\,\dot{x}_0^2\,dt+\int_{t_1}^{t_2}d_5\,\dot{\theta}_1^2\,dt=\int_{t_1}^{t_2}K_m V\dot{x}_0 dt \quad (27)$$

Using this energy equation, the parameter identification was performed by the non-negative least square minimization method in MATLAB®.

If one considers the $K_m$as a parameter to be identified, the numerical optimization procedure, used for finding the optimum parameters would lead to all-zero solution. That is the trivial solution. The reason is as there is no constant number on the right-hand side of the set of equations, the procedure would find the optimum least square minimization of [A]{d}=0, which is trivially zero. In order to avoid numerical complexity, different values of K_m = [0.6 0.7 0.8 0.9] was chosen to find the parameters and the set with minimum error was introduced as real parameters. The system was excited from resting position, fully extended downward configuration, with 4Volts of input and the position and angle of the pendulum was stored.

IV. STATE SPACE AND LQR CONTROLLER

The state space equations of a system of cart and inverted pendulum were adopted from work done by Bogdanov [5]. The Lagrangian equations of motion were re-written as:

$$\boldsymbol{D}(q)\ddot{q} + \boldsymbol{C}(q,\dot{q})\dot{q} + \boldsymbol{G}(q) = \begin{pmatrix} K_m \\ 0 \end{pmatrix} V_{in} \quad (28)$$

$$\boldsymbol{D}(q) = \begin{bmatrix} d_1 & d_2\cos\theta_1 \\ d_2\cos\theta_1 & d_4 \end{bmatrix} \quad (29)$$

$$\boldsymbol{C}(q,\dot{q}) = \begin{bmatrix} -d_2\sin(\theta_1)\dot{\theta}_1 & -d_3\sin(\theta_2)\dot{\theta}_2 \\ 0 & d_5\sin(\theta_1-\theta_2)\dot{\theta}_2 + d_9 \end{bmatrix} \quad (30)$$

$$\boldsymbol{G}(q) = \begin{pmatrix} 0 \\ -d_2\sin\theta_1 \end{pmatrix} \quad (31)$$

In order to re-arrange equations, into a canonical state-space representation of the systems a state vector of X was defined.

$$x = \begin{pmatrix} x \\ \theta_1 \\ \dot{x} \\ \dot{\theta}_1 \end{pmatrix} = \begin{pmatrix} \theta \\ \dot{\theta} \end{pmatrix} \quad (32)$$

$$\dot{x} = \boldsymbol{A}x + \boldsymbol{B}u \quad (33)$$

To alleviate the nonlinearity of the system and considering that the system is about to work at near zero-point state, the matrices were linearized using the Taylor's series expansion of sine and cosine functions and higher order was considered to be zero. Equations of linearized D, C and G matrices at zero neighborhood were formed and using the linearized matrices, A and B matrices were derived.

$$A = \begin{pmatrix} 0 & I \\ -\boldsymbol{D}(0)^{-1}\dfrac{\partial \boldsymbol{G}(0)}{\partial\theta} & -\boldsymbol{D}^{-1}(0)\boldsymbol{C}(0) \end{pmatrix} \quad (34)$$

$$B = \begin{pmatrix} 0 \\ \boldsymbol{D}(0)^{-1}\begin{pmatrix} K_m \\ 0 \end{pmatrix} \end{pmatrix} \quad (35)$$

Where I is the 2x2 identity matrix.

Linear Quadratic Regulator (LQR) method was implemented to stabilize single pendula in upright position. As a necessary condition of the stability in the LQR controller design, the input to the system, u, should be in the form of u = −Kx, where K is the vector of LQR coefficients. In order to find K, the built-in function of lqr(A, B, Q, R) in MATLAB® was used. As per syntax of this function, A and B are the state space equation coefficient matrices while Q and R, are arbitrary error minimizing and controller effort minimizing weights.

V. SWING UP CONTROLLER

In order to swing up the single pendulum from the resting position, the energy method introduce by [15] was adopted. This approach is essentially to minimize the energy of the pendulum while swinging over the unstable up-right position.

The energy of the pendulum at any position is defined as:

$$E(\theta) = \frac{1}{2}J\dot{\theta}^2 + mgl(\cos(\theta) - 1) \quad (36)$$

It reveals that at the upright position, θ = 0, the potential energy term vanishes and the controller tries to put efforts as much as needed to equate the energy to zero. This will lead to incremental increase of the kinetic energy in a way that the total energy at upright position equates zero. By derivation of energy equation with respect to time, it can be proved that the energy of system is increasing as far as $\dot{\theta} \neq 0$ and $\cos(\theta) \neq 0$ and the control law be used as below [15].

$$u(\theta) = V_0(E(\theta) - E(0))sign(\dot{\theta}\cos(\theta)) \quad (37)$$

Where V0 is the maximum voltage equals to 6. In terms of switching from swing up controller to LQR controller two approach were implemented. In the first approach, titled as "non-weighted control transition", a logical comparator was implemented to switch the controller from swing-up to LQR when the pendulum angle passes in ±10 degree around the up-right position. To alleviate abrupt switch between controllers, a control transition zone, titled as "weighted control transition", could be achieved of high between ±45 degrees around up-right position. Within the transition zone a weighted linear equation transiting from 100% swing-up signal at ±45 degree to 100% LQR signal at ±10 degree was defined. Equations 38 and 39 defines the control rules of a single pendulum from resting position to full stance at upright position.

$$\begin{cases} u = -Kx & \theta < 10° \\ = wV_0\big(E(\theta) - E(0)\big)sign\big(\dot{\theta}\cos(\theta) - 1\big) - (1-w)Kx & 10° < \theta < 45° \\ u = V_0\big(E(\theta) - E(0)\big)sign\big(\dot{\theta}\cos(\theta) - 1\big) & \theta > 45° \end{cases} \quad (38)$$

$$w = \frac{1}{(45° - 10°)}(\theta - 10°) \quad (39)$$

In order to perform the simulation of the controlled system, a SimMechanics® mechanical model of the system was constructed for stabilization and swinging up. The physical values of the system parameters were calculated from the identified parameters of the system. For experimental setup, the Quanser® single pendulum system on a cart was operated and it is shown in fig. 2 [16].

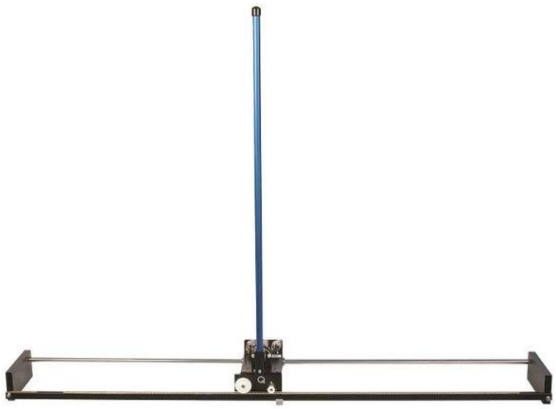


Fig 2: Quanser® single inverted pendulum system

VI. RESULTS AND DISCUSSION

Fig 3 depicts the velocity of the cart along with the 30 points moving average filtered data just as a sample of data which was used for parameter identification.

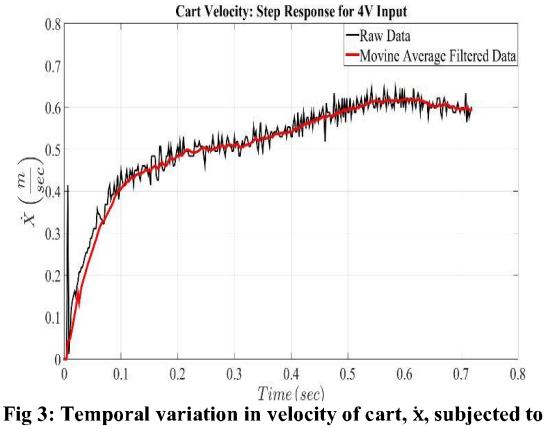


Fig 3: Temporal variation in velocity of cart, ẋ, subjected to step 4V excitation.

The motor constant of 0.7 could result in parameter-set with minimum error which were implemented for controller design. Table I summarizes the results of parameters. It is obvious that since the effect of the rotational friction on the performance of the system was small, the least square algorithm was unable to identify its effects and returned $d_5 = 0$

Table I: Parameters obtained for single pendulum system

K_m	d_1	d_2	d_3	d_4	d_5
0.7	0.528	0.0601	0.0253	4.2290	0

The LQR controller constant was obtained using parameters identified. By testing different values of Q and R, the system response was improve in terms of controller effort and error. The stabilization of the single pendulum with LQR controller constants of K = [-50, -51.4, 181, 38.4] for different initial conditions were captured from experiments (see fig. 4). The experimental results showed a fair agreement with the simulation results and comparison is plotted in fig. 5. Fig 6 shows the recovery of system from multiple environmental (impact) disturbance.

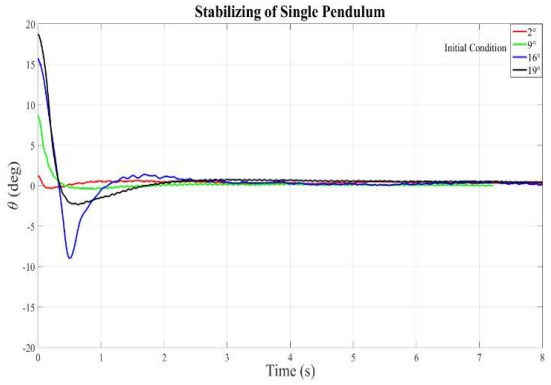


Fig 4: Experimental stabilization and recovery of single inverted pendulum from different initial conditions.

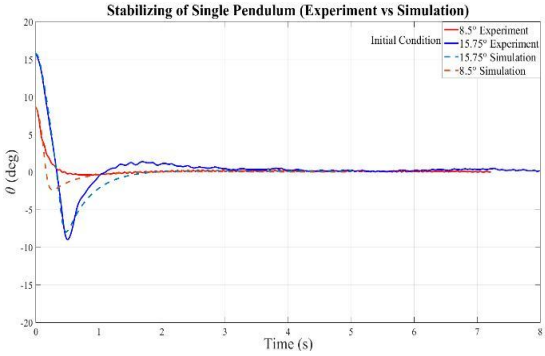


Fig 5: Comparison of experimental and simulated stabilization and recovery of single inverted pendulum from 8.5 and 15.75 degree of initial condition.

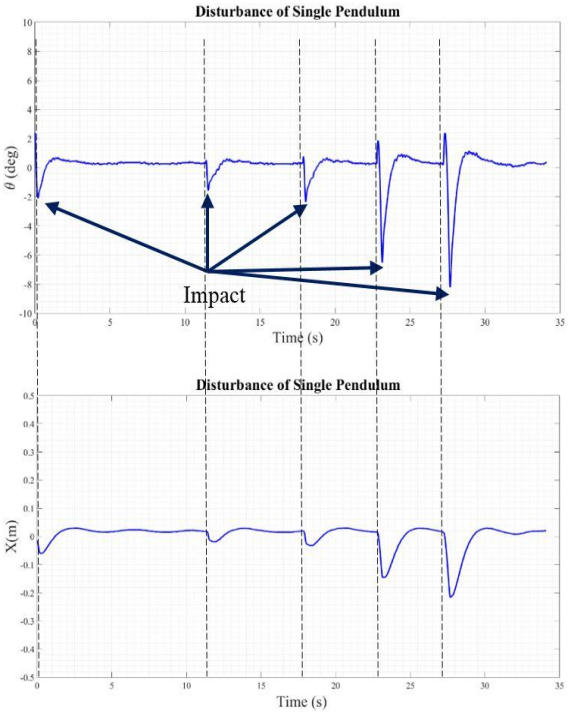


Fig 6: Stabilization and recovery of single inverted pendulum from disturbance by applying impact.

The experimental result for swing up controllers and its transition to LQR with both strategies, weighted and non-weighted, are shown and compared in fig. 7. The transition part is enlarged and it can be noted that there is an overshot in non-weighted method which

could lead to a jerky movement of the cart (see fig. 7 (b)) and further would complicate the stabilization of the pendulum. On the other hand, the jerky movement could restrict the range of switching between controllers.

The comparison of simulation and experimental data for weighted swing up transition to LQR are shown in fig. 8. In addition, a comparison of total controller effort is presented in fig. 9. It can be perceived that the total controller effort was abruptly changed due to spontaneous switching between LQR and swing-up controllers in non-weighted strategies.

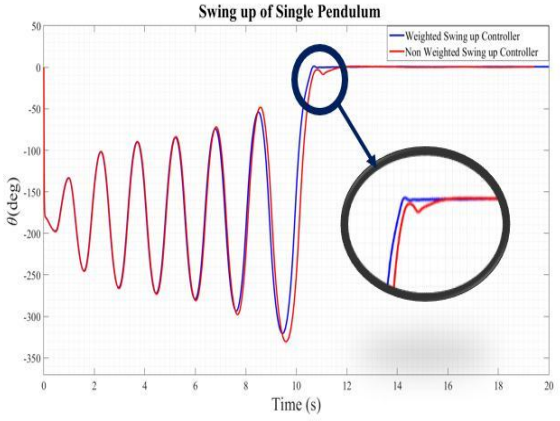


Fig 7: Experimental results of weighted and non-weighted control transition strategies, (a) angle of pendulum and (b) position of cart on rail.

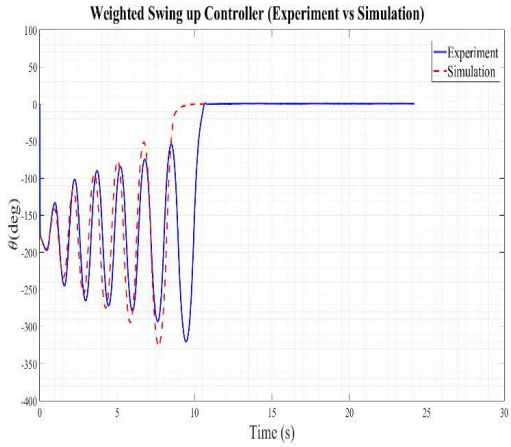


Fig 8: Experimental and simulation results of weighted control transition strategies.

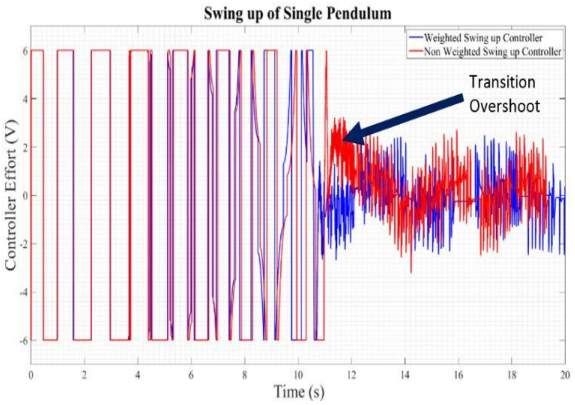


Fig 9: Total controller effort for experimental results of weighted and non-weighted control transition strategies.

CONCLUSION

In this study the swing up of a single inverted pendulum followed by a smooth transition to stabilization by optimal control was performed. Using the system response subjected to 4 Volts excitation, the parameter of system were identified including the viscos friction of cart and rack. The LQR controller designed by using the identified parameters could properly stabilize pendulum without any extra fluctuating. An energy based strategy was presented to swing up the pendula from its rest position in downside and a weighted transition function to LQR controller was proposed. Experimental and modeling results could successfully prevent the jerk in pendulum motion and prevent the overshoot during transition with weighted function in comparison to an abrupt switch.

REFERENCES

[1] S. Jadlovská. Inverted pendula modeling and control. 2009.

[2] M. Nour, J. Ooi and K. Chan. Fuzzy logic control vs. conventional PID control of an inverted pendulum robot. Presented at Intelligent and Advanced Systems, 2007. ICIAS 2007. International Conference On. 2007, .

[3] F. Teng. Real-time control using matlab simulink. Presented at Systems, Man, and Cybernetics, 2000 IEEE International Conference On. 2000, .

[4] Y. Shaoqiang, L. Zhong and L. Xingshan. Modeling and simulation of robot based on matlab/SimMechanics. Presented at Control Conference, 2008. CCC 2008. 27th Chinese. 2008, .

[5] A. Bogdanov. Optimal control of a double inverted pendulum on a cart. Oregon Health and Science University, Tech.Rep.CSE-04-006, OGI School of Science and Engineering, Beaverton, OR 2004.

[6] W. Zhong and H. Röck. Energy and passivity based control of the double inverted pendulum on a cart. Presented at Control Applications, 2001.(CCA'01). Proceedings of the 2001 IEEE International Conference On. 2001, .

[7] L. Fang, W. J. Chen and S. U. Cheang. Friction compensation for a double inverted pendulum. Presented at Control Applications, 2001.(CCA'01). Proceedings of the 2001 IEEE International Conference On. 2001, .

[8] T. Žilić, D. Pavković and D. Zorc. Modeling and control of a pneumatically actuated inverted pendulum. ISA Trans. 48(3), pp. 327-335. 2009.

[9] J. Moreno-Valenzuela, C. Aguilar-Avelar, S. Puga-Guzmán and V. Santibáñez. Two adaptive control strategies for trajectory tracking of the inertia wheel pendulum: Neural

networks vis à vis model regressor. Intelligent Automation & Soft Computing pp. 1-11. 2016.

[10] L. Cervantes and O. Castillo. "Control problem and proposed method," in Hierarchical Type-2 Fuzzy Aggregation of Fuzzy ControllersAnonymous 2016, .

[11] E. Aranda-Escolástico, M. Guinaldo, M. Santos and S. Dormido. Control of a chain pendulum: A fuzzy logic approach. International Journal of Computational Intelligence Systems 9(2), pp. 281-295. 2016.

[12] H. Khebbache, M. Tadjine and S. Labiod. Adaptive sensor-fault tolerant control for a class of MIMO uncertain nonlinear systems: Adaptive nonlinear filter-based dynamic surface control. Journal of the Franklin Institute 2016.

[13] Z. Gan, T. Wiestner, M. A. Weishaupt, N. M. Waldern and C. D. Remy. Passive dynamics explain quadrupedal walking, trotting, and tölting. Journal of Computational and Nonlinear Dynamics 11(2), pp. 021008. 2016.

[14] M. Gautier and W. Khalil. On the identification of the inertial parameters of robots. Presented at Decision and Control, 1988., Proceedings of the 27th IEEE Conference On. 1988, .

[15] K. J. Åström and K. Furuta. Swinging up a pendulum by energy control. Automatica 36(2), pp. 287-295. 2000.

[16] Quanser Consulting Inc., "Linear Servo Base Unit with Inverted Pendulum," vol. 2016, 2016.

★★★