A NEW FUZZY BASED CONTROLLER DESIGN TO ACHIEVE SYNCHRONIZATION BETWEEN TIME-VARYING FRACTIONAL-ORDER CHAOTIC SYSTEMS

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Abstract- Synchronization of time-varying fractional unified chaotic systems is studied, in this paper. Parameter of an unified chaotic system plays an important role in chaotic systems. Also orders of systems have undeniable effect on fractional-order chaotic systems. We consider parameter and orders of this system as a time-varying system. synchronization of two different fractional order unified chaotic systems with a controller which is based on Lyapunov stability theory. Numerical simulation is shown to verify the results.

Index Terms- Synchronization, Time-varying parameter , Unified chaotic system , Fuzzy based controller.

I. INTRODUCTION

Fractional calculation is generalization of integer calculation. Many systems are modeled with fractional-order dynamics [1-3]. Existence of chaos in fractional order systems is studied by Grigorenko and Grigorenko in 2003, for the first time [4]. After that many other fractional order chaotic systems are introduced, such as fractional order Chua's system [5], fractional order Lorenz system [6], fractional order Chen system [7], fractional-order financial system [8]. One of the most popular fractional chaotic systems is the fractional unified chaotic system [9].

Unified chaotic system is a chaotic system which changes between Lorenz, Lü and Chen chaotic systems family with parameter changes.

Chaotic systems have wide range of applications in many research field and one of these applications is chaos synchronization.

Many effective methods have been introduced to achieve the synchronization between fractional-order chaotic systems, such as adaptive synchronization [10], observer-based synchronization [11], backstepping control [12], nonlinear feedback synchronization [13] and optimal synchronization [14]. Also there are some studies on synchronization of fractional order unified chaotic systems such as [9],[22]. These methods are used for certain fractional order unified systems. In this paper we introduce a method for synchronization of time-varying fractional unified chaotic system. These variations are on parameter and orders of systems. For simulations of the proposed method, we consider two different fractional time-varying chaotic systems and design a suitable controller based on Lyapunov method and fuzzy logic for synchronization of two systems.

II. FRACTIONAL CALCULATION

A. FRACTIONAL OPERATOR DEFINITIONS

There are some definitions for fractional order derivate [15,16]. Three most popular definitions are described in follows.

1) Grünwald-Letnikov fractional derivative

$$D_t^{\alpha} f(t) = lim_{h\to 0}\, h^{-\alpha} \sum_{h=0}^{t-a/h} (-1)^j \binom{\alpha}{j} f(t-jh) \quad (1)$$

In Grünwald -Letnikov (GL) definition, fractional order derivate is defined as

where $\binom{\alpha}{j} = \frac{\alpha(\alpha-1)\ldots(\alpha-j+1)}{j!}$.

2) Riemann-Liouville fractional derivative

Riemann-Liouville (RL) definition of fractional order derivate is given as follows.

$$D^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha+n+1}}\, d\tau \quad (2)$$

where n is an integer such that n-1 <α < n, and Γ is the Gamma function.

3) Caputo fractional derivative

Caputo definition of fractional order derivate is described as

$$D^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}}\, d\tau \quad (3)$$

Like RL definition, n is an integer such that n-1 <α < n, and $ Γ is the Gamma function.

B. Stability of fractional order systems

Consider the following fractional order equation

$D^{\alpha} x = f(x)$ (4)

where $x = [x_1, x_2, \ldots, x_n]^T$ are the states of system and

$f(x) = [f(x_1), f(x_2), \ldots, f(x_n)]^T$ describe the system's equations. Also we suppose $0 < \alpha \leq 1$.

For this condition we have the following Lemma.

Lemma1.

$J = \partial f(x)/\partial x$ is the Jacobian matrix at the

equilibrium points of system (4).

If all the eigenvalues of J satisfy the following condition[17,18].

$$|arg(spec(J))| > \frac{\alpha\pi}{2} \quad (5)$$

Where α is fractional order of system.

Then, system (4) is locally asymptotically stable.

Integer version of system (\ref{eq4}) is described as

$$\frac{dx}{dt} = f(x) \quad (6)$$

Remark1.

According to Lemma 1, for integer order system(6), stability condition is determined as

$$|arg(spec(J))| > \frac{\alpha\pi}{2} \quad (7)$$

Corollary 1.

According to Remark 1, for $0 < \alpha \leq 1$, stability region for system (4) contains system(6) stability region.

Corollary 1 shows that for a fractional order system is stable when integer version of system is stable. Then we can achieve stability of a fractional order system from integer version of system[19,20].

II. SYSTEM DESCRIPTION

For first time Lü et al. introduced a kind of chaotic systems which has some similarity to three types of well-known chaotic systems[21].Unified chaotic system is a system which switch between Lorenz , Lü and Chen systems. They showed that these systems can be wrote in a unified form. They suggested this unified form as follows.

$$\begin{cases} \dot{x} = (25\beta + 10)(y - x) \\ \dot{y} = (28 - 35\beta)x - xz + (29\beta - 1)y \\ \dot{z} = xy - \dfrac{(8+\beta)x}{3} \end{cases} \quad (8)$$

Where x, y, z are the states and $\beta \in [0,1]$ is the parameter of system which switches the behavior of system. When $\beta \in [0,0.8]$, it behaves as general Lorenz system. When $\beta = 0.8$ system behaves as chaotic Lü system and when $\beta \in [0.8,1]$ system is like chaotic Chen system.

Fractional order unified system was introduced in [22]. This system is descried as follows.

$$\begin{cases} D^{\alpha}x = (25\beta + 10)(y - x) \\ D^{\alpha}y = (28 - 35\beta)x - xz + (29\beta - 1)y \\ D^{\alpha}z = xy - \dfrac{(8+\beta)x}{3} \end{cases} \quad (9)$$

Ref.[22] shows that fractional order unified system behaves chaotic when $0.91 < \alpha \leq 1$. Chaotic attractors of system(9) for $\alpha = 0.95$ is shown in Fig.\ref{fig1}. Like integer order unified system, fractional order unified system has different behavior for different values of β. Then for each $\beta \in [0,1]$ system(9) has chaotic behavior with $0.91 < \alpha \leq 1$.

In this paper, drive and response systems are time-parameteras a continuous variable parameter on the interval $\beta \in [0,1]$. Also we suppose that fractional order of system is a continuous random variable with uniform distribution on the interval [0.91,1].

For this case we show the chaotic behavior of fractional order unified system in Fig1 .

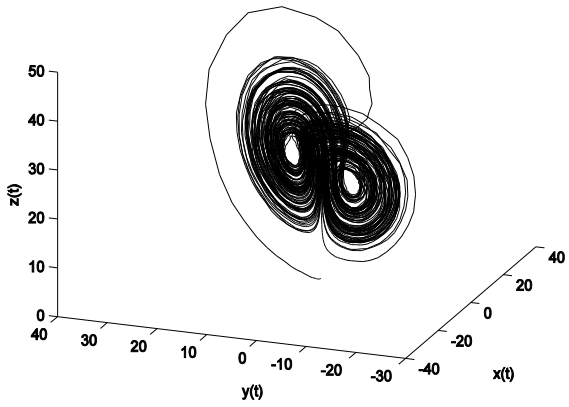


Fig 1. Chaotic attractor of fractional-order unified system with time varying parameter and order

III. SYNCHRONIZATION METHOD

In this section, synchronization of two fractional order unified systems is proposed. In synchronization of chaotic systems, there are two systems, a drive system and a response system as seen in the following.

$$D^{\alpha}X_d = f(X_d) \quad (10)$$

$$D^{\alpha}X_r = f(X_r, U) \quad (11)$$

Where X_d is the drive system state vector, X_r is the response system state vector and U is control signal vector which is designed to converge synchronization error to zero.

In this paper we consider two different fractional order chaotic systems with time-varying parameter and orders.

Difference of two systems is considered on time-varying parameter of systems.

We consider two different time-varying parameters in two systems (β and γ) where these parameters are bounded in [0,1].

Orders of two systems are considered identical orders which randomly variable with time between 0.91 and 1. We write these systems as follows.

$$\begin{cases} D^{\alpha}x_d = (25\beta + 10)(y_d - x_d) \\ D^{\alpha}y_d = (28 - 35\beta)x_d - x_d z_d + (29\beta - 1)y_d \\ D^{\alpha}z_d = x_d y_d - \dfrac{(8+\beta)x_d}{3} \end{cases} \quad (12)$$

$$\begin{cases} D^{\alpha}x_r = (25\beta + 10)(y_r - x_r) + u_1' \\ D^{\alpha}y_r = (28 - 35\beta)x_r - x_r z_r + (29\beta - 1)y_r + u_2' \\ D^{\alpha}z_r = x_r y_r - \dfrac{(8+\beta)x_r}{3} + u_3' \end{cases} \quad (13)$$

Define a new parameter as

$\theta = \gamma - \beta$

Then we can write synchronization errors between two systems as

$$\begin{cases} D^{\alpha}e_1 = (25\beta + 10)(e_2 - e_1) + 25\theta\,(y_r - x_r) + u_1' \\ D^{\alpha}e_2 = (28 - 35\beta)e_1 + x_r e_3 - z_d e_1 + (29\beta - 1)e_2 \\ \qquad -35\theta\, x_r + 29\theta\, y_r + u_2' \\ D^{\alpha}e_3 = x_r e_2 + y_d e_1 - \dfrac{(8+\beta)e_3}{3} - \dfrac{\theta}{3} z_r + u_3' \end{cases} \quad (14)$$

Where $e_1 = x_r - x_d$, $e_2 = y_r - y_d$ and $e_3 = z_r - z_d$ are the synchronization errors.

We can write integer version of Eq.(\ref{eq19}) as

$$\begin{cases} \dot{e_1} = (25\beta + 10)(e_2 - e_1) + 25\theta\,(y_r - x_r) + u_1' \\ \dot{e_2} = (28 - 35\beta)e_1 + x_r e_3 - z_d e_1 + (29\beta - 1)e_2 \\ \qquad -35\theta\, x_r + 29\theta\, y_r + u_2' \\ \dot{e_3} = x_r e_2 + y_d e_1 - \dfrac{(8+\beta)e_3}{3} - \dfrac{\theta}{3} z_r + u_3' \end{cases} \quad (15)$$

with remove nonlinear parts as a part of controller, we have

$$\begin{cases} \dot{e_1} = (25\beta + 10)(e_2 - e_1) + u_1 \\ \dot{e_2} = (28 - 35\beta)e_1 + (29\beta - 1)e_2 + u_2 \\ \dot{e_3} = -\dfrac{(8+\beta)e_3}{3} - \dfrac{\theta}{3} z_r + u_3 \end{cases} \quad (16)$$

A Lyapunov functions as

$V = e_1^2 + e_2^2 + e_3^2$

To apply the fuzzy Lyapunov based method, we assume the equations(16) are unknown and we have the following partial knowledge about the equations:

1. The relevant state variables are $e_1 = 0$, $e_2 = 0$ and $e_3 = 0$.
2. $\dot{e_1}$ is proportional to u_1 , that is, when u_1 increases (decreases) $\dot{e_1}$ increases (decreases).
3. $\dot{e_2}$ is proportional to u_2 , that is, when u_1 increases (decreases) $\dot{e_2}$ increases (decreases).
4. $\dot{e_3}$ is proportional to u_3 , that is, when u_3 increases (decreases) $\dot{e_3}$ increases (decreases).

Obviously we assume very little knowledge indeed. Our objective is to design u_1, u_2, u_3 that $(0,0,0)$ would be a stable point of (16). We consider above Lyapunov function candidate.

Now for $\dot{V} < 0$ we must have have

$$\dot{V} = 2e_1\dot{e_1} + 2e_2\dot{e_2} + 2e_3\dot{e_3} < 0 \quad (17)$$

$$\dot{V} = (25\beta + 10)(e_2 - e_1)e_1 + 2e_2\dot{e_2} - \frac{8+\beta}{3}e_3^2 < 0 \quad (18)$$

We can now derive sufficient conditions for (17) to hold:

If e_1 is positive and e_2 is positive, Then u_2 is negative big.

If e_1 is positive and e_2 is negative, Then u_2 is zero.

If e_1 is negative and e_2 is positive, Then u_2 is zero.

If e_1 is negative and e_2 is negative, Then u_2 is negative big.

We require that $\dot{V}$ be negative, it is natural to examine the signs of e_1 and e_2, so the obvious fuzzy partition is {positive, negative}. The partition for $\dot{e_2}$, namely {negative big, zero, positive big} was obtained similarly when we plugged the linguistic values {positive, negative} for e_1 and

e_2 in (18). To ensure that $\dot{V} < 0$ is satisfied even though we do not know e_1 and e_2 exact magnitudes, only that it is positive (negative), we must set $\dot{e_2}$ to negative big (positive big).

Following [8], we characterize the linguistic variables positive, negative, negative big, zero and positive big by the membership functions

$$\begin{aligned} \mu_{+}(e) &= \frac{1}{1 - exp(-30e)} \\ \mu_{-}(e) &= \frac{1}{1 + exp(-30e)} \\ \mu_{-big}(u_2) &= exp(-(5 + u_2)^2) \\ \mu_{0}(u_2) &= exp(-u_2^2) \\ \mu_{+big}(u_2) &= exp(-(5 - u_2)^2) \end{aligned} \quad (19)$$

and using the center of gravity defuzzifier and the product inference engine, we obtain the following stabilizing controller:

$$u_2 = \; 5f_1(e_1, e_2) \mid f_2(e_1, e_2)$$

where

$$f_1 = \frac{\mu_{+}(e_1)\mu_{+}(e_2)}{\mu_{+}(e_1)\mu_{+}(e_2) + \mu_{+}(e_1)\mu_{-}(e_2) + \mu_{-}(e_1)\mu_{+}(e_2) + \mu_{-}(e_1)\mu_{-}(e_2)}$$

and

$$f_2 = \frac{\mu_{-}(e_1)\mu_{-}(e_2)}{\mu_{+}(e_1)\mu_{+}(e_2) + \mu_{+}(e_1)\mu_{-}(e_2) + \mu_{-}(e_1)\mu_{+}(e_2) + \mu_{-}(e_1)\mu_{-}(e_2)}$$

IV. SIMULATION RESULTS

In this section, to verify and demonstrate the effectiveness of the proposed method, we simulate the synchronization problem for two different fractional order unified systems with an example.

In this example two fractional order unified systems are considered as drive and response systems. Orders of systems are selected as a random order with uniform distribution on the interval $[0.91,1]$, for drive and response systems. Time-varying parameters are selected as random and different parameters with uniform distribution on the interval $[0,1]$. Also initial conditions are selected as

$(x_r(0), y_r(0), z_r(0)) = (2,2,3)$

and $(x_d(0), y_d(0), z_d(0)) = (0.1,0.2,0.3)$.

The result of proposed method is shown in Fig.2 . Also synchronized states are illustrated in Fig.3,4,5.

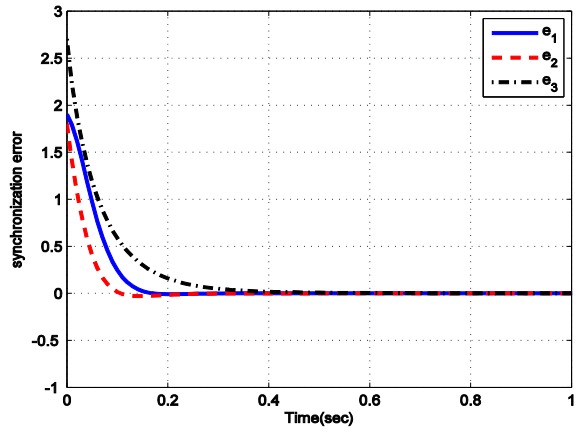


Fig 2. Synchronization errors for two different fractional order unified system with time-varying parameters and orders

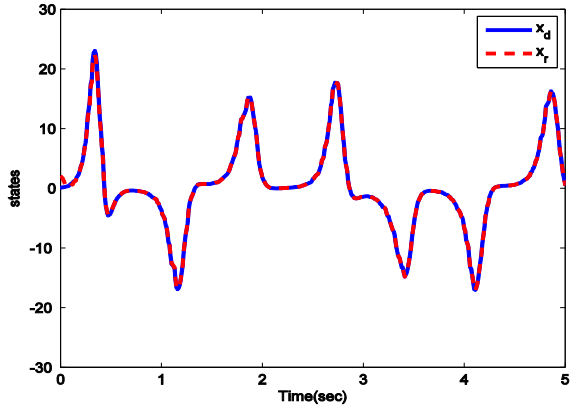


Fig 3. Synchronized states of two systems, x_d, x_r

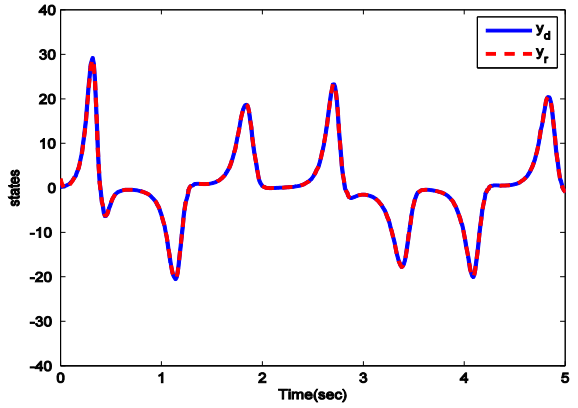


Fig 4. Synchronized states of two systems, y_d, y_r

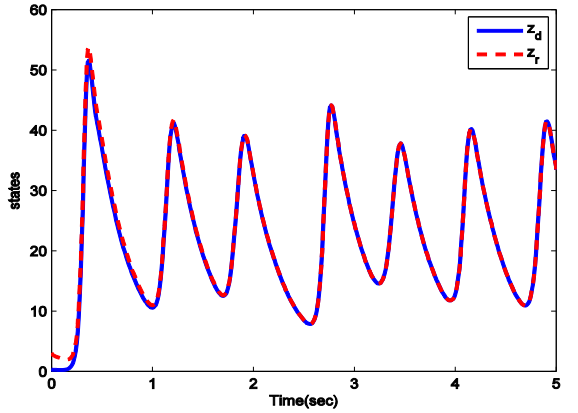


Fig 5. Synchronized states of two systems, z_d, z_r

CONCLUSION

In this paper synchronization of fractional order unified systems is discussed. The proposed method is used for synchronization of two fractional order unified systems with time-varying parameter and orders. We define control inputs for synchronization problem of two different fractional order unified systems. An example is simulated to show the performance of the proposed method.

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