TRANSIENT ANALYSIS OF BRIDGES UNDER IMPULSE LOADING USING THE FINITE PRISM FORMULATION

[1]OMAR MOHAMMAD YOUNIS, [2]AYAD THABET SAEED ALGHABSHA

[1]Faculty of Engineering, Universiti Putra Malaysia, Serdang, Selangor, 43400, Malaysia
[2]Civil Engineering and Architecture School, Chongqing Jiaotong University, Chongqing, 400074, China
E-mail: [1]engomar1976@gmail.com, [2]atsaeed75@yahoo.com

Abstract— In the present work the finite prism formulation was employed for the transient analysis of bridges under suddenly applied pulse loading. In this method a combinationof the finite element method representing the cross section of the prism and Fourier series suitably chosen to represent the behavior of prism in the longitudinal direction (which satisfies the simply supported boundary conditions at the ends), is used. Explicit time integration scheme was used for solving the equation of motion for the bridge. In the present work the damping was neglected in the formulation of the equation of motion of bridges. This render is used to avoid the solution of global system of equations, because each equation becomes uncoupled with other equations. One of the main aspects in the present work is the coupling of the explicit solution technique of the equation of motion with the harmonic solution, using the finite prism method for the problem of bridges under suddenly applied pulse loading. The present results agree closely with the results available in the literature for plates.

Keywords— Finite prism Method, Transient Analysis, Impulsive Loads, Sudden Loads, Pulse Shape, Pulse Duration, Explicit Time Integration Scheme.

I. INTRODUCTION

The purpose of this work is to develop a procedure for obtaining the dynamic response of bridges subjected to impulsive loads. Naturally bridges are three-dimensional structures. The finite prism method was first developed by Zienkiewics and Too [18] for the static analysis of bridges and also developed for the free vibration analysis of straight and curved deck and box girder bridges by AL-Darzi [1]. Saeed and Xiang [12] developed and employed the finite prism method for the dynamic analysis of coupling vehicle-bridge system. The same method is developed and used in the present work for the first time for the dynamic analysis of bridges due to impulsive loadings. In this method a combination of the finite element method representing the cross section of the prism and Fourier series suitably chosen to represent the behavior of the prism in the longitudinal direction, which satisfies the simply supported boundary conditions at the ends is used. Moreover, the three dimensional problem is reduced to a series of two-dimensional problems, which are decoupled for each harmonic number. The explicit time integration scheme is used for solving the equation of motion for the bridge. These afford avoiding the solution of global system of equations, because each equation becomes uncoupled with other equations. One of the main aspects of the present work, is the coupling of the explicit solution technique of the equation of motion with harmonic solution using the finite prism method for the problem of bridges under impulsive loads. The various kinds of pulse shapes in-plane impulsive loads have been used. The validity of the developed numerical model has been studied by analyzing structures such as simply supported plates subjected to impulsive load. The comparison with the available reported results has shown a significant matching.

II. FINITE PRISM FORMULATIONS

$$\{q\} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} \tag{1}$$

where u, v and w are the displacements along the three dimensional Cartesian axes X, Y and Z respectively as shown in Fig. 1:

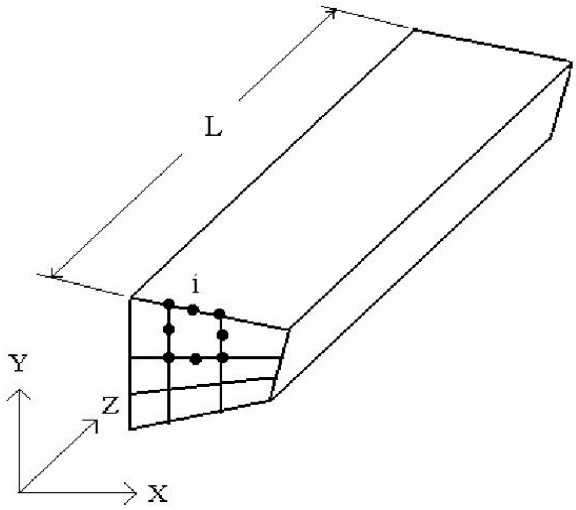


Fig.1. Typical prism element

The displacement at any point in the eight-node prism element can be interpolated as follows [12,1,18]:

$$u(r,s,z) = \sum_{l=1}^{\infty}\sum_{i=1}^{ne} N_i(r,s)u_i^l sin\frac{l\pi z}{L} \tag{2}$$

$$v(r,s,z) = \sum_{l=1}^{\infty}\sum_{i=1}^{ne} N_i(r,s) v_i^l sin\frac{l\pi z}{L}$$

$$w(r,s,z) = \sum_{l=1}^{\infty}\sum_{i=1}^{ne} N_i(r,s) w_i^l sin\frac{l\pi z}{L}$$

where u_i^l, v_i^l and w_i^l are the nodal line displacements amplitudes for harmonic number l at node i for prism e, ne is the total number of nodal lines for the prism which is equal to eight, N_i is the interpolation (shape) function in terms of the local coordinates r, s which vary from -1 to $+1$ within the cross section of the prism.

The discretized expression for strains and stresses within an element can be expressed as [13]:

$$\{\varepsilon\} = \sum_{l=1}^{\infty}\sum_{i=1}^{ne}[B_i^l]\{d_i^l\} \quad (3)$$

$$\{\sigma\} = [D]\sum_{l=1}^{\infty}\sum_{i=1}^{ne}[B_i^l]\{d_i^l\} \quad (4)$$

where $[D]$ is the elasticity matrix [15] and the strain-displacement matrix $[B_i^l]$ associated with nodal line i for the harmonic l can be expressed as [1]:

$$[B_i^l] = \begin{bmatrix} \frac{\partial N_i}{\partial x}Sl & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y}Sl & 0 \\ 0 & 0 & -\frac{l\pi}{L}N_iSl \\ \frac{\partial N_i}{\partial y}Sl & \frac{\partial N_i}{\partial x}Sl & 0 \\ \frac{l\pi}{L}N_iCl & 0 & \frac{\partial N_i}{\partial x}Cl \\ 0 & \frac{l\pi}{L}N_iCl & \frac{\partial N_i}{\partial y}Cl \end{bmatrix} \quad (5)$$

$$\{d_i^l\} = [u_i^l, v_i^l, w_i^l]^T \quad (6)$$

The strain and stress components in vector form can be written as:

$$\{\varepsilon\} = \{\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{xz}, \gamma_{yz}\}^T \quad (7)$$

$$\{\sigma\} = \{\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{xz}, \tau_{yz}\}^T \quad (8)$$

III. DYNAMIC EQUATIONS

The dynamic equation of motion of bridge can be expressed as:

$$[M]\{\ddot{d}\} + [C]\{\dot{d}\} + [K]\{d\} = \{F_t\} \quad (9)$$

where $[M]$, $[C]$ and $[K]$ are the global mass, damping and stiffness matrices respectively and $\{\ddot{d}\}$, $\{\dot{d}\}$ and $\{d\}$ are the acceleration, velocity and displacement vectors respectively. $\{F_t\}$ is the external applied load, which is function of time t.

The damping matrix can be written as [17]:

$$[C] = \int_v [N]^T c\,[N] dv \quad (10)$$

Due to the orthogonality properties [1] i.e.;

$$\left.\begin{aligned} \int_o^L sin\frac{l\pi z}{L}.sin\frac{m\pi z}{L}dz \\ \int_o^L cos\frac{l\pi z}{L}.cos\frac{m\pi z}{L}dz \end{aligned}\right\} \begin{aligned} &= \frac{L}{2} whenl = m \\ &= 0 \ whenl \neq m \end{aligned} \quad (11)$$

The stiffness and mass matrices for harmonic $l$can be written as:

$$[K_{ij}^{ll}] = \frac{L}{2}\iint_A [B_i^l]^T [D][B_j^l]\,dx.dy \quad (12)$$

$$[M_{ij}^{ll}] = \frac{L}{2}\iint_A [N_i^l]^T \rho [N_j^l]\,dx.dy \quad (13)$$

The equivalent nodal force vector can be expressed as:

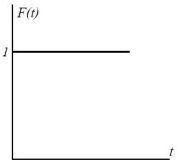
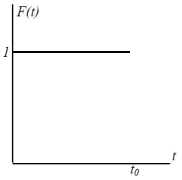
$$\{F_t\} = \sum_{l=1}^{\infty}\sum_{i=1}^{ne}\begin{bmatrix} N_i.Sl & 0 & 0 \\ 0 & N_i.Sl & 0 \\ 0 & 0 & N_i.Cl \end{bmatrix}\{f_p\} \quad (14)$$

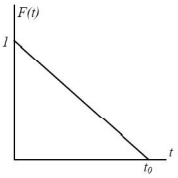
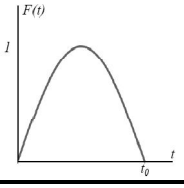
The integration of stiffness and mass matrices are carried out numerically using Gauss quadrature method.

IV. IMPULSIVE LOADS

Various kinds of pulse shapes of impulsive loads are used in this work. The impulsive load $q(x,y,t) = q_0F(t)f(x,y)$ is applied on the top surface of the bridge, in which q_0 is the maximum amplitude, $f(x,y)$ is a unit fuction in space domain and $F(t)$is a unit function in time domain which can be any one of the kinds listed in Table 1.

Table 1: The Various kinds of pulse shapes of impulsive loads

Case	F(t)
1	 $F(t) = 1$ Sudden loads
2	 $F(t) = \begin{cases} 1, & t \leq t_0 \\ 0, & t > t_0 \end{cases}$ Step loads

3		$F(t) = \begin{cases} 1 - \frac{t}{t_0}, & t \le t_0 \\ 0, & t > t_0 \end{cases}$ Triangular Loads
4		$F(t) = \begin{cases} sin\frac{\pi t}{t_0}, & t \le t_0 \\ 0, & t > t_0 \end{cases}$ Sine loads

V. SOLUTION OF THE EQUATION OF MOTIOM

The integration of the equation of motion (Eq. (9)) is carried out using the explicit central difference scheme [8,9]. This is only applicable when the mass matrix is lumped mass matrix. This requires the transformation of constant mass matrix in Eq. (13) into a lumped mass matrix as in the following.

5.1. Mass Lumping

The most simple method of lumping the prism mass matrix is by adding all terms of each line of the consistent mass matrix and placing the result on the diagonal, such a process gives the nodal masses in the following form [2,5]:

$$\bar{M}_{ii}^{e} = \sum M_{ij}^{e} \tag{15}$$

This method fails when applied to eight-node isoparametric element. The reason is that negative nodal masses are obtained in corner nodes. To overcome this, several alternative schemes have been investigated in the literature [5]. In the present work a new approach was developed in which the total mass of the element is divided equally between mid- side and corner nodes as follows:

$$\bar{M}_{ii}^{e} = \frac{1}{8} M^{e} \tag{16}$$

In another approach suggested by Owen and Hinton and by others [4,5,6,9], the mass is lumped to the diagonal terms of the consistent mass matrix, as follows:

$$\left.\begin{aligned} \bar{M}_{ii}^{e} &= M_{ii}^{e}\left(\frac{M^{e}}{\sum M_{ii}^{e}}\right) \\ M_{ii}^{e} &= \int_{v} N_i^2 \rho dv \end{aligned}\right\} \tag{17}$$

where M^e is the total mass of the element. This procedure is also used in the present work and both approaches gave almost same results.

VI. EXPLICIT TIME INTEGRATION SCHEME

The dynamic equation of motion for harmonic $l$can be written at time t as:

$$[M]\{\ddot{d}_t\}^l + [C]\{\dot{d}_t\}^l + \{P_t\}^l = \{F_t\}^l \tag{18}$$

where $\{P_t\}^l$ is the global vector of internal resisting nodal forces and $\{F_t\}^l$ is the vector of equivalent nodal forces resulting from the impulsive load $q(x, y, t)$.

In the central difference approximation, the acceleration can be written as follows:

$$\{\ddot{d}_t\}^l = \frac{1}{(\Delta t)^2}[\{d_{t+\Delta t}\}^l - 2\{d_t\}^l + \{d_{t-\Delta t}\}^l] \tag{19}$$

and the velocity may also be expressed as follows:

$$\{\dot{d}_t\}^l = \frac{1}{2(\Delta t)}[\{d_{t+\Delta t}\}^l - \{d_{t-\Delta t}\}^l] \tag{20}$$

Substituting Eq. (19) and Eq. (20) in Eq. (18) and solving for $\{d_{t+\Delta t}\}^l$ yields:

$$\begin{aligned} \{d_{t+\Delta t}\}^l = &\left([M] + \frac{\Delta t}{2}[C]\right)^{-1} \\ &\Big[(\Delta t)^2\{R_t\}^l + 2[M]\{d_t\}^l \\ &- \left([M] - \frac{\Delta t}{2}[C]\right)\{d_{t-\Delta t}\}^l\Big] \end{aligned} \tag{21}$$

where $\{R_t\}^l$ is the residual force vector at time *t*, which is equal to:

$$\{R_t\}^l = \{F_t\}^l - \{P_t\}^l \tag{22}$$

If the matrices $[M]$ and $[C]$ are both diagonal matrices then the solution of Eq. (21) for degree of freedom *i* can be easily written as follows:

$$\begin{aligned} (d_i)_{t+\Delta t}^l = &\left(m_{ii} + \frac{\Delta t}{2}c_{ii}\right)^{-1}\Big[(\Delta t)^2(rf_i)_t \\ &+ 2m_{ii}(d_i)_t^l \\ &- \left(m_{ii} - \frac{\Delta t}{2}c_{ii}\right)(d_i)_{t-\Delta t}^l\Big] \end{aligned} \tag{23}$$

where $(d_i)_{t+\Delta t}^l$ is the displacement in the i^{th} degree of freedom for harmonic number l. m_{ii} and c_{ii} are the corresponding diagonal terms of the mass and damping matrices, and rf_i is the corresponding component of residual force vector.

6.1. Starting Algorithm

Since the governing equilibrium Eq. (23) or Eq. (21) involves information at the previous time steps t and $t - \Delta t$, a starting algorithm is therefore required. Based on the initial conditions, the value at $0 - \Delta t$ may be obtained from Eq. (20).

$$\left(\dot{d}_i\right)_0^l = \frac{1}{2(\Delta t)}\left[(d_i)_{0+\Delta t}^l - (d_i)_{0-\Delta t}^l\right] \tag{24}$$

from which, we get:

$$(d_i)^l_{0-\Delta t} = -2\Delta t(\dot{d_i})^l_0 + (\dot{d_i})^l_{0+\Delta t} \quad (25)$$

Substituting Eq. (25) in Eq. (23), the displacements of the i^{th} degree of freedom at the first time step Δt for harmonic l can be evaluated as follows:

$$(d_i)^l_{\Delta t} = \frac{(\Delta t)^2}{2m_{ii}}(rf_i)^l_0 + (d_i)^l_0 + \Delta t\left[1 - \frac{c_{ii}\Delta t}{2m_{ii}}\right](\dot{d_i})^l_0 \quad (26)$$

It is reported in the literatures [3,7], that the damping effect of the bridge is small and can be neglected in the analysis. The explicit algorithm is conditionally stable and it requires small time steps for accurate and stable solution. In the present work the estimation of critical time step given by Tsui and Tong [16] was adopted.

6.2. Dynamic Condition at any Point

The vertical displacement at any point $d^p_{t+\Delta t}$ can be interpolated from the nodal displacement amplitudes at time step $t + \Delta t$, i.e. $\{d_{t+\Delta t}\}^l$ by using Eq. (2). Also, the velocity at any point at time step $t + \Delta t$ can be determined by backward finite difference scheme using the point displacement at time steps t and $t + \Delta t$ as [13]:

$$\dot{d}^p_{t+\Delta t} = \frac{1}{\Delta t}\left[d^p_{t+\Delta t} - d^p_t\right] \quad (27)$$

VII. VALIDATION OF THE METHOD

7.1. NUMERICAL EXAMPLE 1

In order to confirm the accuracy of the developed numerical model, a nondimensional simply supported plate at four edges under suddenly applied partially distributed load (load case 1) was first considered, then the plate cross section is divided into five prism elements as shown in Fig. 2. The information in Table 2 gives details of the material properties and the impulsive load. The analysis is carried out using five harmonic terms only.

Table 2: Properties of thick plate material and impulsive load

Item	Value
Length (a)	$\sqrt{2}$
Width (b)	1
Thickness (h)	0.2
Modulus of elasticity (E)	1
Mass density	1
Poisson's ratio	0.3
Uniformly distributed load (q_0)	1

As part of the validation of the present method, The dynamic response of this plate under a sudden load is presented in term of the nondimensionalized centre displacements as shown in Fig. 3. The present results are compared with the finite element results of Reddy [10] using his computing data [10,14]. The present finite prism solution for the centre displacement is in excellent agreement with the finite element solution of Reddy.

The nondimensionalized time ($\bar{t}$) and centre displacement ($\overline{w}$) those are mentioned in the Fig. 3 are defined as $t/b\sqrt{(E/\rho)}$ and $wEah/q_0b^3$ respectively.

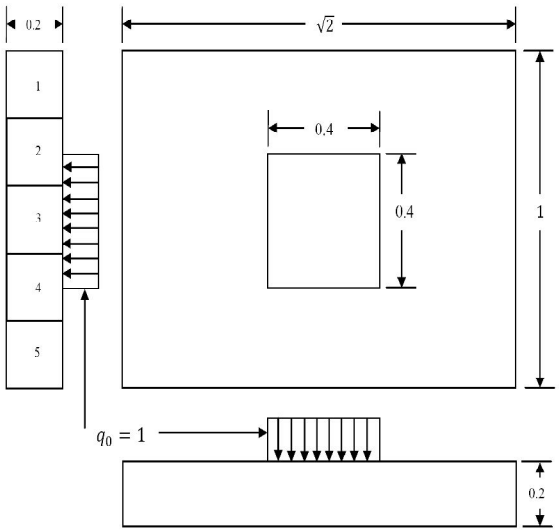


Fig.2. Geometry and finite prism idealization of four opposite edges simply supported plate

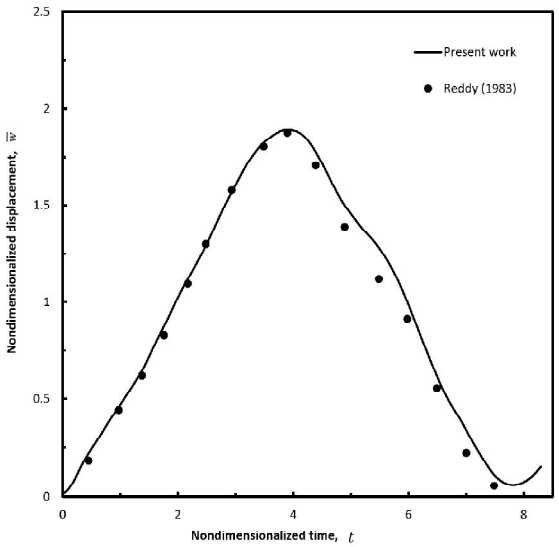


Fig.3. Nondimensionalized centre displacement for simplysupported plate

7.2. NUMERICAL EXAMPLE 2

A simply supported plate with the geometry (see Fig. 4) was subjected to suddenly applied partially distributed load (load case 1) and has been analyzed. The plate cross section is divided into five prism elements. The information in Table 3 gives details of the material properties and the impulsive load.

Table 3: Properties of thin plate material andimpulsive load

Item	Value
Length (a)	1
Width (b)	$\sqrt{2}$
Thickness (h)	0.1
Modulus of elasticity (E)	1
Mass density	1
Poisson's ratio	0.3
Uniformly distributed load (q_0)	1

The analysis is carried out using five harmonic terms only. The nondimensionalized centre displacement of the rectangular plate subjected to a sudden load are calculated and compared in the Fig. 5 with modal superposition approach solution of Reismann and Tendorf [11] using their computing data [11,14]. The present results are quite accurate and in excellent agreement with the solution of Reismann and Tendorf.

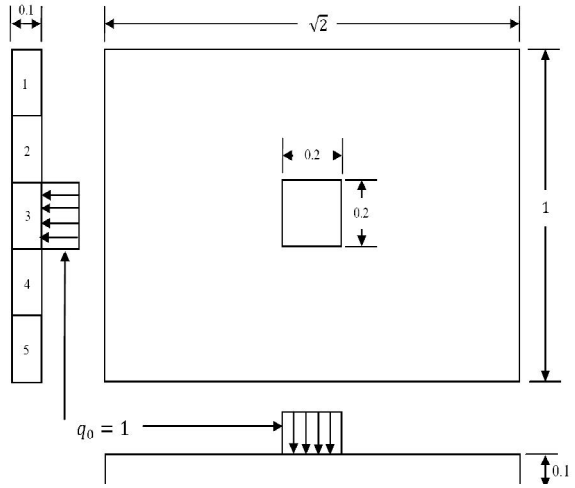


Fig.4. Geometry and finite prism idealization of simply supported plate

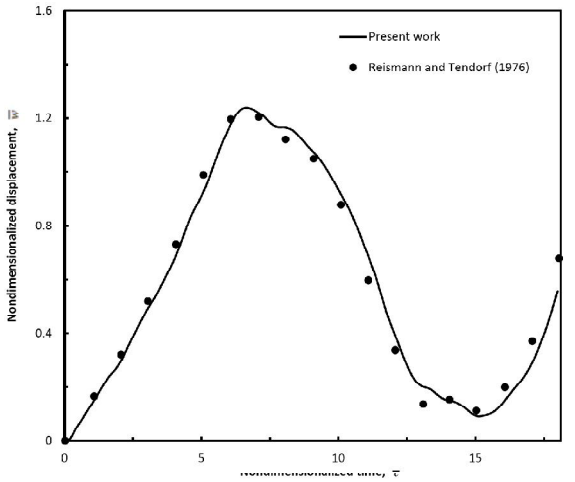


Fig.5. Nondimensionalized centre displacement for simply supported plate

VIII. DYNAMIC ANALYSIS OF DECK GIRDER BRIDGE

A simply supported deck girder bridge with the geometry shown in Fig. 6, was subjected to sudden and impulsive loads. The bridge cross-section is divided into 25 prism elements as shown in Fig. 7. The information in Table 4 gives details of the material properties and the impulsive load.

Table 4: Properties of bridge material and impulsive load

Item	Value
Modules of elasticity	200000 MPa (Concrete)
Mass density	2400 kg/m^3
Span length	15 m
Poisson's ratio	0.2
Load	200 kN

Figs. 8 and 9 show the plots of the centre normalized displacement for the above problem (i.e. simply supported deck girder bridge) under the loading condition of cases 1-4, i.e., sudden loads, step loads, triangular loads and sine loads in Table 1, when the applied load was removed at t=0.002 sec and t=0.004 sec (i.e. pulse duration) respectively. The normalized displacement at mid span is defined as "the ratio of the dynamic centre displacement to the maximum static centre displacement". The analysis is carried out using five harmonic terms only. It is noticed that the maximum normalized displacement takes place in the case sudden loading, and the dynamic response behavior of the bridge are significantly influenced by the pulse shape of impulsive load.

Fig. 10 shows the effect of the pulse duration on the dynamic response of the deck girder bridge under the loading of cases 1 and 2, when the applied load was removed at t=0.003 sec, t=0.006 sec and t=0.012 sec. It is clear that, the dynamic response behavior of the bridge are significantly influenced by the pulse duration of impulsive load, and when the pulse duration increases, the dynamic response completely deviate from the static one.

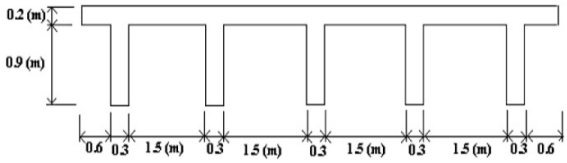


Fig.6. Details of cross-section of deck girder bridge

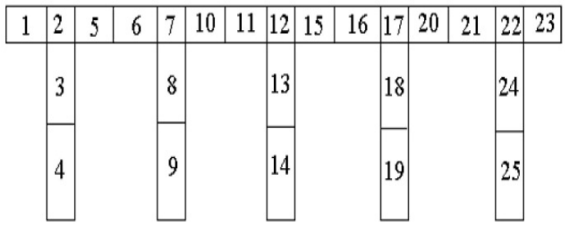


Fig.7. Finite prism idealization of deck girder bridge

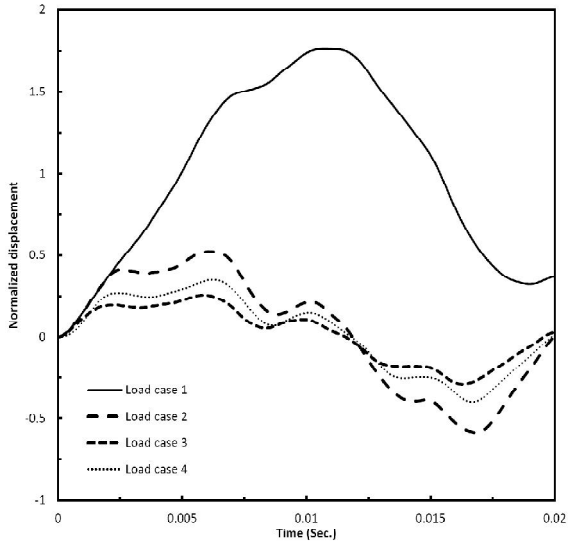


Fig.8. Effect of pulse shape on dynamic behaviors of the deck girder bridge(pulse duration = 0.002 Sec)

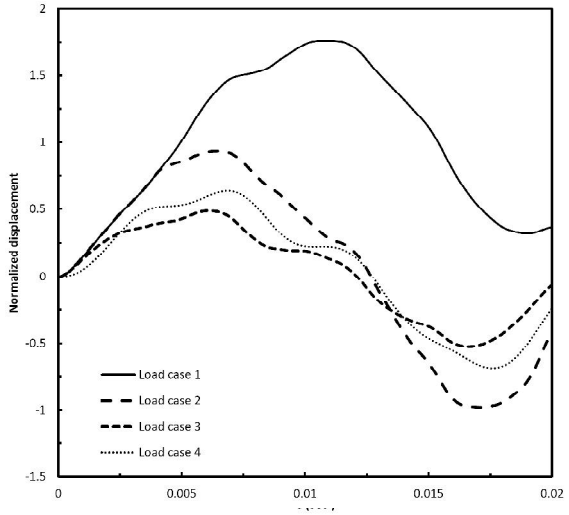


Fig.9. Effect of pulse shape on dynamic behaviors of the deck girder bridge(pulse duration = 0.004 Sec)

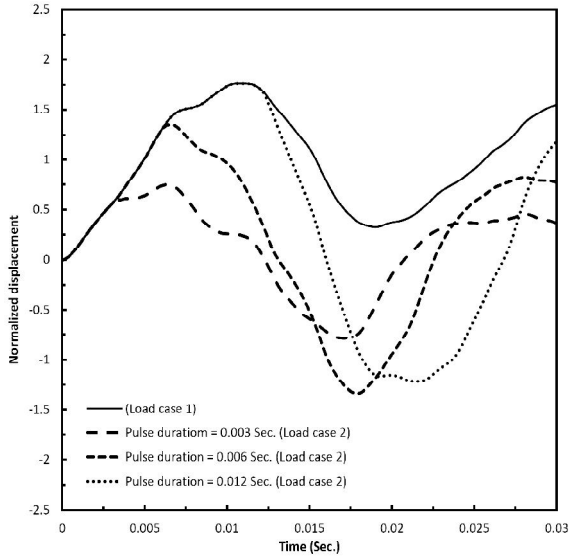


Fig.10. Effect of pulse duration on dynamicbehaviors of the deck girder bridge subjected to step load

CONCLUSIONS

A new model for the transient analysis of bridges due to sudden and impulsive loads was developed in this work. The model is based on the coupling of the explicit solution of the equation of motion with the harmonic solution of the finite prism method.

The numerical results from the developed computer code in the present study using finite prism method have shown excellent agreement with published data. A parametric study of deck girder bridge subjected to sudden and impulsive loads was carried out. The Various kinds of pulse shapes of impulsive loads were unused in this study. The problem of sudden and impulsive loads on the bridge shows that the dynamic response behavior of the bridge are significantly influenced by the pulse shape and pulse duration of impulsive load.

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