TIME INDEXED SCHEDULING OF SINGLE PRODUCTION LINE TO MINIMIZE THE TOTAL PRIORITIZE CUSTOMER TARDINESS IN CARPET INDUSTRY

NADER AL THEEB

Industrial Engineering Department, Jordan University of Science & Technology, Jordan
E-mail: naaltheeb@just.edu.jo

Abstract - In this research, an optimization model is developed to determine the best schedule in a factory specialized in carpet production. The developed model considers most of realistic considerations, such as taking the prioritization of customer tardiness, but not the products. Different data sets were taken from the demand records in the factory and solved by CPLEX- Concert Technology. The results shows that schedules resulted from the model save up to 4.4% of total prioritized tardiness compared with traditional methods that were used in the factory in most of cases. Some better results, suchas22.2%ofimprovementisachievedinadata-set, where more investigation is required to analyze such results.

Keywords – Production Scheduling;

I. INTRODUCTION

Theproblemtobe consideredinthisresearch istheschedulingof carpetproductionina singleline factory.The
main aim of the study in to minimize the total weighted (prioritized) tardiness of customers;but not the

jobs.Asthisproblemconsistsasetofcustomersj∈{1,2,3,...M},asMisthenumberofcustomers.Eachcustomerhas apriorityvaluedependsontheamountofprofitcomesfrom itandbasedonthelongtermcontractssignedwith
the factory.Additionally, each customer has a preferred due date.Customers order different types of

products with differentquantities,i∈{1,2,3,...n},as Nis the number of products.For example, a specific product i could be ordered by three customers with different quantities.

Once the production line is set up to produce a product, the whole requested quantity should be produced, even thisquantitymightberequestedbycustomerswithhighpriorityandbyothercustomerswithlesspriority. Thereason thatthewholequantityofeachproductshouldbeproduced inonerun,evenitisrequestedbydifferentcustomers,is to eliminate the effect long set up times.Each customer receives its requests in a single delivery date to reduce the shipping cost.So, the delivery date for each customer is the latest day of the last days of production of all request products by the customer.Accordingly, the challenge in this problem is how to schedule the production of different products, so the total weighted tardiness of customers to be minimum.

II. RELATED LITERATURE

Scheduling is a critical part in production.Thus, it has attracted many researcher to work in this field.Most of research, thathasbeendoneinthefieldofsingleproductionlinewith anobjectivetominimizethetotaltardinessor weighted tardiness, was to develop solution approaches because of NP-hard nature of such problems.For example,
BierwirthandKuhpfahl(2017)havedevelopedaheuristic approachtominimizethetotalweightedtardinessofjob shop scheduling problem.Pessoa et al. (2022) have utilized the column generation algorithm to minimize the total weightedtardinessforsinglemachineproblem. JaramilloandErkoc(2017)havedevelopedmulti-stagesalgorithmto
minimizethetotaltardinessandovertimecostsforthesche dulingofsinglemachineGonza´lezandVela(2015)have suggested memetic algorithm to solve the single machine scheduling with setups.
Research related to single machine scheduling regard the customers are limited.For example, Geramipour et al.
(2017)havetriedtomaximizethetotalprofitcomesfromc ustomerorderswithtardinesspenalties. Kimetal.(2017) have developed a framework to schedule the single machine with possibility of order cancellation if the tardiness exceedsapre-determinedallowabletime.Thecloserresearchtothisstud yhasbeendonebyRasti-BarzokiandHejazi (2013) who have developed a model to minimize the total number of tardy jobs; these jobs are ordered by different customers,andeachcustomerhasonlyonedeliverydate.
Asnoted,thereisnostudytocovetheminimizationoftotal weighted tardiness of customers who order common multiple products.

III. PROBLEM DESCRIPTION AND MATHEMATICAL FORMULATION

The problem under consideration involves a set of products, set of customers with different priority, set of time slots defining the working days.Each customer requests specific demand from each type of

carpet, customers have different priority based on delay penalty and the long term contracts signed with the factory.When the productionis changed from product to other, long setup is needed. Thus, once the production line starts producing a product type i, the whole requested quantity from this type should be produced even it is requested by different customers withdifferentprioritiesandduedates.Thistechniqueisutilizedtominimizethesetuptimethatisneededtoswitch between production different products.

The objective of the proposed model is to determine the optimal scheduling to produce the products that are requested by different customers, so the total weighted tardiness is minimum.

3.1. Notation-Parameters

In this subsection, the sets and parameter notations are presented as follows.The time horizon Tis partitioned intodiscreteintegerperiodswithequal length, such that t={1,2,...,T}.There are other two sets to define the productsi= {1,2,...,P},andtodefinethecustomersj={1,2,...,C},wherePandC arethenumberofproductsand customersrespectively.Eachcustomerhasapriorityvalue$P_j$toreflectitsimportancetofactorybasedontheexpected profitthatcouldbederivedfromitandthepenaltycostthatcouldbepaidincaseoflatedelivery. Additionally,each customerhasanexpecteddeliverydateD_j.Thequantityof productirequestedbycustomerjisdefinedbyparameter q_{ij},where $Q_i$is the total quantity of product i requested by all customers.If this total quantity $Q_i$is divided by the production capacity; the result will represent the total number of days (TD_i) required to produce this product.Next table summarizes the aforementioned sets and parameters.

Notation	Description
C	Setofcustomers j∈{0,1,2,...,C}
P	Setofproducts i∈{0,1,2,...,P}
T	Setoftimeslotst∈{0,1,2,...,T}
P_j	Priorityofcustomerj
D_j	Expectedduedateofcustomerj
q_{ij}	Requesteddemandoftypeibycustomerj
Q_i	Totalrequesteddemandoftypeibyallcustomers
TD_i	Totalnumberofdaysrequiredtoproducethewholerequesteddemandofproducti

Table1: Summary of parameternotation

3.2. Notation-DecisionVariables

Differentdecisionvariablesareusedtodefinethesolution providedbytheconstructedmodel. FirstX_{it}isabinary variablesetdefinedtodetermineiftheproductiisproduced inthetimeslottornot. Second,variablesDel_jrepresent thedeliverytimeforcustomerj,whichisthelastdaythatlastproductrequestedbycustomerjisproduced.Third,YL_i andYS$_i$arethelatestandtheearliesttimesandthattheproductiisstartedtobeproducedandfinished,respectively. Orderof each customer can be delayed from the expected due date by a value called tardiness T_j.The definitions of all decision variables are presented in the next table.

Notation	Description
X_{it}	Binaryvariable=1iftheproductiisproducedattimet,0otherwise
Del_j	Thetimethatthecustomerjwillreceiveitsorder
T_j	Thetardinessofthecustomerj
YL_i	Thelastdaythattheproductiisproduced
YS_i	Thefirst(beginning)daythattheproductiisproduced

Table2:Summary of decision variables

3.3. Mathematical Model

The MILP model of this problem is divided into eight constraints, as follows

$$Min \quad \sum_{j=1}^{C} P_j \times T_j \tag{1}$$

$$\sum_{t=1}^{T} X_{it} = TD_i \quad \forall\, i \in P \tag{2}$$

$$YS_i \le t \times X_{it} - (M \times (1 - X_{it})) \quad \forall\, i \in P,\ t \in T \tag{3}$$

$$YL_i \ge t \times X_{it} \quad \forall\, i \in P,\ t \in T \tag{4}$$

$$\sum_{i=1}^{P} X_{it} \le 1 \quad \forall\, t \in T \tag{5}$$

$$\sum_{i=1}^{T} X_{it} = YL_i - YS_j + 1 \quad \forall\, i \in P \tag{6}$$

$$Del_j \ge YL_i \forall\, i \in P,\ j \in C : q_{ij} > 0 \tag{7}$$

$$T_j \ge Del_j - D_j \forall\ j \in C \tag{8}$$

$$T_j \ge 0 \forall\ j \in C \tag{9}$$

$$T_j,\ Del_j,\ YL_i,\ YS_i \ge 0 \text{ and integer} \tag{10}$$

$$X_{it} \ge 0 \text{ and binary} \tag{11}$$

Theobjectivefunction(1)istominimizethesummationof prioritymultipliedbythetardinessforallcustomers. Constraints (2) ensure that the total number binary variables with value of one, which represent that the product is produce in day t, equals to the number of days that are required to produce product i.Constraints (3) define that the starting day to produce product i which equals to the least time of all binary variables equal to one for this product; the second term in the right hand side of this constraint is to make sure that this constraint is applied when only the binary variable is one.Similarly, constraints (4) define that the last day of product i production which equals to the last time of all binary variables equal to one for this product.Constraints (5) ensure that only one product can be produced at day t.Constraints (6) ensure that once the product i production is started, it should continue until the whole required quantity is produced with irruption; such that the total number of binary variables equal to one is the differencebetweenthelastandstartdaysofproduction. Constraints(7)specifythatthedeliverydayforthecustom er jisthelatestdayoffinishproductiondaysforallrequestedp roductsbythiscustomers. So,thisconstraintisapplied foracustomeronlyforrequestedproduct,suchthatifitdoe snotorderaproduct;thefinishtimeofthisproductisnot counted.Constraints (8) define the tardiness of jobj which equals to the delivery date minus the expected due date. Constraints(9)definethatthetardinessisonlypositive value, becausetheearlydeliveryhasnobenefit.Constraints (10)and(11)definethevariables.

IV. NUMERICAL ANALYSIS

In this section, numerical experiments on various data sets are performed using CPLEX within a two-hour time limittoevaluatetheeffectivenessofthesuggestedmathem aticalmodel.Dataweretakenfromthefactory,eachdata set includes the orders per month, quantities of each product that was ordered by each customer, the actual schedule that was performed at that month. The number of data sets (months) will be considered in this study is eight sets.
CPLEX was able in two hours limit to provide optimal solution for four data sets, and near optimal for the other fourdatasets.Basedontheavailableactualschedulethatw ereperformed;theobjectivefunctionvaluesarecalculate d according to the tardiness and customer priorities.Priorities have been determined by the factory management to minimize the penalty costs and keep the customer satisfaction high for the most of customers.

The next table shows the actual objective function values and the resulted objective function values (optimized) provided by the model.

Dataset(Month)	OFVprovidedbythemodel	ActualOFV
1	29409	29409
2*	5925	6061
3*	12555	13104

4*	7168	7168
5	11090	11330
6*	5844	5844
7	19715	19802
8	24511	29964

*DenotethattheOFVisoptimal

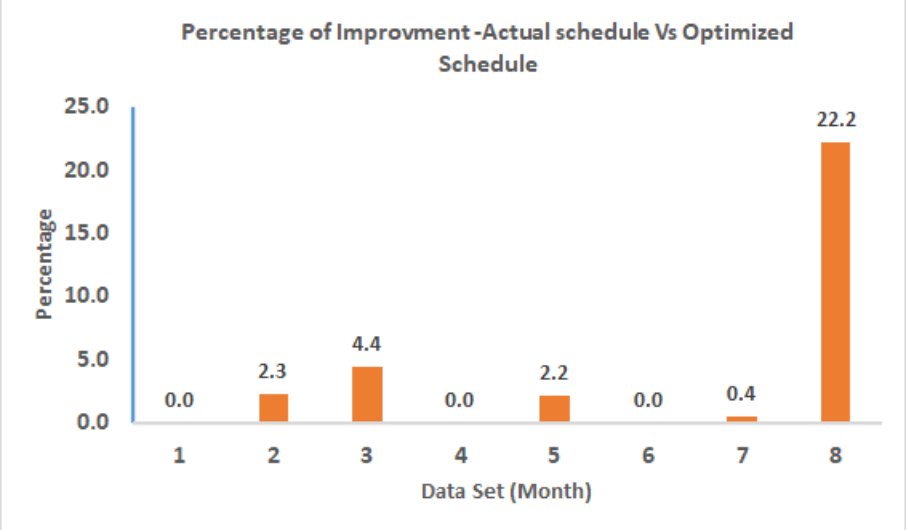
Table3: ObjectiveValuesforactualandoptimizedresults

Tomakemoreclearcomparison,thepercentageofimprovement(POI)iscalculatedasinthenextequation.

$$Percentage\ of\ Improvement = \frac{ActualOFV - OptimizedOFV}{ActualOFV} \times 100\% \quad (12)$$

The next figure shows the POI for all of the data sets.It can be seen that the model provide similar schedules to theactualschedulesinthreemonthswith POI = 0. Thisisexpectedinmonthswithlowdemandrequestedwithfew customers order in that month.In this case, there is possibility to achieve an excellent schedule when depending on experienceandsomemakesenseprinciples. Theothernotethatthemodelisabletoprovidesolutionsbetterthanthe actual solutions with POI ranging from 0.4% to 4.4% in other 4 months.This happen in months with high demand orders with various product and high number of customers where is finding good solutions based on experience in veryhard.Thelastnotethatthemodelprovideasolutionforadatasetbetterthantheactualsolutionby22.2%,which isahugeimprovement.Thismighthappeniftheactualplansarereallydonebypersonswithnoorshortexperiences.

The main issue in these results is that the model is always capable to provide results better or equal to the results achieved without depending on optimization models. This could lead to huge economic saving in long terms.



V. CONCLUSION

Inthisresearch,anoptimizationmodelisdevelopedtooptimizetheschedulingofcarpetindustry. Itisfoundthat the results provided by the model can defeat the results of actual plans by a POI up to 5% in most of cases.In some easycases,actualplansaresimilartotheoptimizedplans,andsomecomplicatedcases;optimizedplanscanbebetter than the actual plan with POI greater than 20%.

Thisresearchcanbeextendedbytakingmorethan8dataset swithlongerperiodthanamonth. Additionally,data sets with unexpected results, such as POI greater than 20%, can be investigated more to take the advantage of such improvements.

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