# MODAL ANALYSIS OF WING SKELETON OF A MAV TO DETERMINE THE RANGE OF FLAPPING FREQUENCIES

**[1]ADITYA RASTOGI, [2]AASTIK CHAWLA**

[1]Delhi Technological University, [2]Delhi Technological University
E-mail: [1]adityarastogi@dce.ac.in, [2]aastikchawla_2k12@dce.edu

**Abstract-** Determining The Natural Frequencies Of Structure Under Time Dependent Cyclic Loading Is Very Important To Control The Amplitude Of Vibration Caused By That Force. If The Frequency Of Force/Load Applied Matches The Natural Frequencies Of The Structure, The Structure Can Fail Owing To Large Deformation And Fatigue. Modal Analysis Using ANSYS Workbench Was Performed On The Wing Skeleton Of A Micro Air Vehicle Which Was Driven By A Four Bar Crank-Rocker Mechanism; Crank Being Driven By A Gear Imparts Cyclic Time Dependent Force On The Rocker Coupled To The Wing. The Natural Frequencies Of The Wing Were Determined And The Flapping Frequency Range For Different Flight Domains Was Restricted On The Basis Of The Result. Using Mechanical Dynamic Analysis Module Of ANSYS The Participation Factor Was Determined To Further Optimise The Structure And Operational Parameters.

**Keywords-** M :- Structure Mass Matrix K :- Structure Stiffness Matrix C :- Damping Coefficient Matrix U:- Displacement Vector U':- Velocity Vector U":- Acceleration Vector F(T):- Applied Force Vector Ωi:- Angular Frequency Of Ith Mode

## I. INTRODUCTION

Based on D'Alemberts principle and due to the discretization process of a continuous structure with finite elements the following equation of motion can be derived:

$$M \cdot u'' + C \cdot u' + K \cdot u = F(T) \quad (1)$$

Dynamical equilibrium is obtained if equation (1) holds for all times t. All problems in structural dynamics can be formulated based on the above equation of motion (1). A coarse classification is obtained by taking different representations for the time varying applied forces. For this classification figure 1 presents a diagram where several analysis types of structural dynamics are listed according to the representation of the applied load[1].

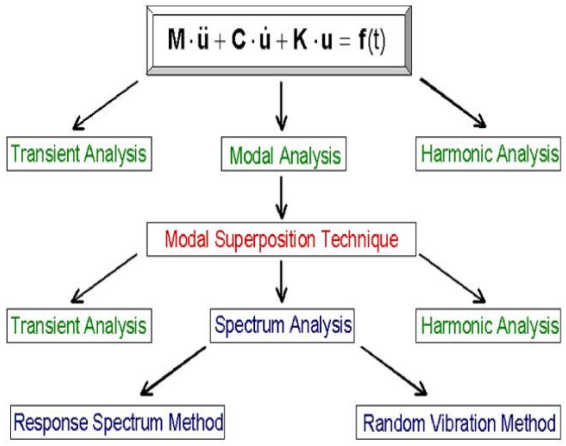



**Fig. 1**

The solution of this nonlinear differential equation (as k can be a function of u) can be made simpler for different conditions by taking suitable assumptions a structural analysis that accounts for damping can be distinguished from an analysis where damping is neglected. Finally, since not every engineering application results in the formulation of symmetric system matrices we should further distinguish between analyses with symmetric and unsymmetric matrices. The modal analysis technique is used to determine the vibration characteristics (i.e., natural frequencies and mode shapes) of linear elastic structures. The most fundamental of all dynamic analysis types, it allows the design to avoid resonant vibrations or to vibrate at a specified frequency. It gives engineers an idea of how the design will respond to different types of dynamic loads and helps in calculating solution controls for other dynamic analyses. In this case an un-damped system with symmetric stiffness matrix is considered. The right hand side of the equation of motion (1) is considered to be zero, i.e. f (t) = 0. A modal analysis can also be taken as a basis for other more detailed dynamic analyses such as a transient dynamic analysis, a harmonic analysis or even a spectrum analysis based on the modal superposition technique. The modal analysis is a linear analysis. Any nonlinearity which may have been specified by the user is ignored. However, pre-stress effects may be considered. More information about the modal analysis using ansys can be found in [2] and [5].

The Linear Differential Equation For Linear, Free, Un-Damped Vibration Motion Is

$$M \cdot u'' + K \cdot u = 0$$

Assuming Harmonic Motion

$$u = \Phi_i \sin(\Omega_i t + \Theta_i)$$

$$u'' = -\Omega_i^2 \Phi_i \sin(\Omega_i t + \Theta_i)$$

This Implies

$$\{K - \Omega_i^2 m\}\{\Phi_i\} = 0$$

For Non- Trivial Solutions to Occur

$$\mathrm{Det}\{K - \Omega_i^2 m\} = 0$$

The solution of this equation is given by the eigen value which gives the square of natural frequency and the corresponding eigen vector gives the mode shape i.e. the shape assumed by the structure when vibrating at frequency f. The Block Lanczos method is a very efficient algorithm to perform a modal analysis for large models [4]. It is a fast and robust algorithm and used for most applications as the default solver [3]. The system's generalized mass matrix 'm' is given by

$$M = \varphi^t \, m \, \varphi$$

Let 'r'

Be the influence vector which represents the displacements of the masses resulting from static application of a unit ground displacement. The influence vector induces a rigid body motion in all modes. Define a coefficient vector

$$L = \Phi^t m \, R$$

The Modal Participation Factor Matrix Γi For Mode I Is

$$\Gamma_i = L_i / M_{Ii}$$

The Effective Modal Mass

$$M_{eff} = L_i^2 / M_{ii}$$

It measures the amount of mass moving in each direction for each mode. A high value in a direction indicates that the mode will be excited by forces in that direction.

Design And Simulation Parameters

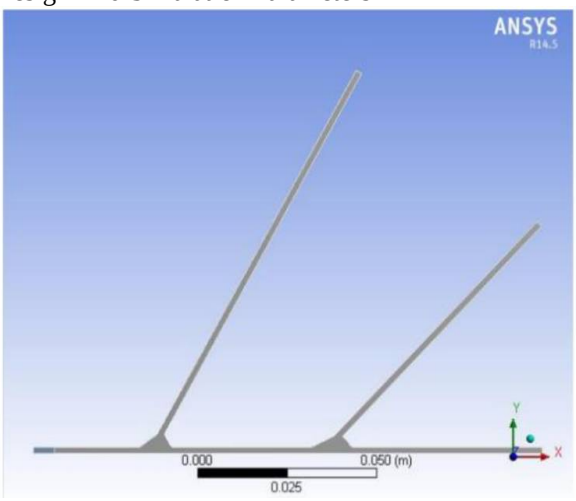



**Fig 2**

| Object Name | *Solidg(In Grey Colour)* | *Solid(In Blue Colour)* |
|---|---|---|
| State | Meshed | |
| **Graphics Properties** | | |
| Visible | Yes | |
| Transparency | 1 | |
| **Definition** | | |
| Suppressed | No | |
| Stiffness Behavior | Flexible | |
| Coordinate System | Default Coordinate System | |
| Reference Temperature | By Environment | |
| **Material** | | |
| Assignment | Veroblue | |
| Nonlinear Effects | Yes | |
| Thermal Strain Effects | Yes | |
| **Bounding Box** | | |
| Length X | 0.136 M | 6.E-003 M |
| Length Y | 0.10127 M | 1.7e-003 M |
| Length Z | 1.7e-003 M | |
| **Properties** | | |
| Volume | 1.0083e-006 M³ | 1.734e-008 M³ |
| Mass | 1.825e-003 Kg | 3.1385e-005 Kg |
| Centroid X | 7.8549e-002 M | 3.E-003 M |
| Centroid Y | 2.3882e-002 M | 0. M |
| Centroid Z | -3.1245e-020 M | 0. M |
| Moment Of Inertia Ip1 | 1.3423e-006 Kg·M² | 1.5117e-011 Kg·M² |
| Moment Of Inertia Ip2 | 2.271e-006 Kg·M² | 1.0171e-010 Kg·M² |
| Moment Of Inertia Ip3 | 3.6124e-006 Kg·M² | 1.0171e-010 Kg·M² |
| **Statistics** | | |
| Nodes | 4241 | 297 |
| Elements | 536 | 40 |
| Mesh Metric | None | |

**Table 1.**

The Fig 2. Shows the design of the wing skeleton , the blue part is clamped in the metallic hinge therefore is assumed to be having zero degree of freedom. Table 1 shows all the physical parameters of the design, the material used is veroblue and the part is 3d printed. Properties of veroblue are shown in table 2.

| Density Kg/M³ | Young's Modulus Pa | Poisson's Ratio | Bulk Modulus Pa | Shear Modulus Pa |
|---|---|---|---|---|
| 1810 | 2.75e+009 | 0.3 | 2.2917e+009 | 1.0577e+009 |

**Table 2**

Solution Options Are:-

Problem Dimensionality. . . . . .3-D

Degrees Of Freedom. . . . . Ux Uy Uz
Analysis Type . . . . . .Modal
Extraction Method. . . . . .Block Lanczos
Offset Temperature From Absolute Zero . . . . . 273.15
Number Of Modes To Extract. . . . . 6
Globally Assembled Matrix . . . . . .Symmetric
Number Of Modes To Expand . . . . . .All

**SIMULATION RESULTS**

FREQUENCIES FROM BLOCK LANCZOS ITERATION
MODE FREQUENCY (HERTZ)
1 10.99001052849
2 11.05725991661
3 19.68172657386
4 24.24582884144
5 38.28729527625
6 62.14671924002

The following bar chart indicates the frequency at each calculated mode.

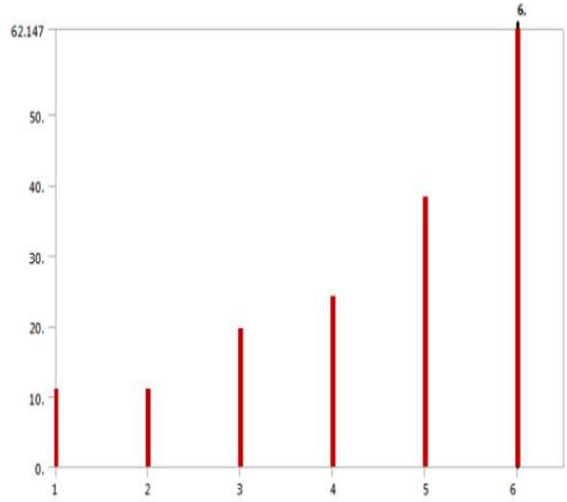



**Fig 3**

| Mode | Frequency | Partic Factor[X] | Partic Factor[Y] | Partic factor[Z] |
|---|---|---|---|---|
| 1 | 10.9900 | -0.14620E-06 | 0.36318E-06 | 0.35865E-01 |
| 2 | 11.0573 | -0.12742E-01 | 0.31612E-01 | -0.41211E-06 |
| 3 | 19.6817 | -0.49692E-09 | 0.84880E-09 | 0.44754E-02 |
| 4 | 24.2458 | 0.97436E-02 | 0.85774E-04 | -0.60480E-09 |
| 5 | 38.2873 | - 0.23591E-09 | 0.85774E-04 | -0.97459E-02 |
| 6 | 62.1467 | 0.12472E-01 | 0.85774E-04 | -0.10142E-09 |

**Table 3 Partic factor for translation along X,Y and Z axis**

| Mode | Frequency | Partic Factor[RotX] | Partic Factor[RotY] | Partic Factor[RotZ] |
|---|---|---|---|---|
| 1 | 10.9900 | 0.11491E-02 | -0.35674E-02 | 0.44877E-07 |
| 2 | 11.0573 | - 0.13229E-07 | 0.40978E-07 | 0.39068E-02 |
| 3 | 19.6817 | 0.13229E-07 | 0.15835E-03 | 0.10808E-09 |
| 4 | 24.2458 | 0.97778E-12 | 0.70574E-10 | - 0.37274E-03 |
| 5 | 38.2873 | 0.28209E-03 | 0.58231E-03 | 0.74588E-11 |
| 6 | 62.1467 | 0.23635E-11 | 0.58786E-11 | 0.49843E-03 |

**Table 4 Partic factor for rotation along X,Y and Z axis**

Higher the Partic factor along a direction for a particular mode implies that when force in that direction is applied, major amount of mass will vibrate with that mode. It can be seen that for mode 1 of frequency 10.9900 Hz , the maximum Partic factor is towards Z axis in translation and X and Y axis for rotation. The minimum flapping amplitude to support the weight of the MAV is little less than 5 Hz, flapping amplitude taken for lift to weight ratio little higher than 1 is 6Hz. As the lifting and driving forces act along Z axis and pitching moment along X axis, the amount of mass vibrating with mode 1 of frequency 10.9900 Hz is maximum, which is very close to our operational flapping frequency, as the frequency of flapping for hovering is higher than for cruising therefore the wings cannot be used in a design meant for hovering.

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