LAMINAR NATURAL CONVECTION OVER A HEATED CYLINDER IN A CUBIC – A LATTICE BOLTZMANN STUDY

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In the present study, the thermal lattice Boltzmann scheme is used for simulating the natural convection in cubical enclosure with different inner cylinder positions at Rayleigh numbers of 10^4- 10^6 and a Prandtl number of 0.7. The location of the inner cylinder is changed vertically along the centerline of the enclosure. The transient state and steady state results are investigated and analyzed. The effects of the boundary wall on heat transfer and fluid flow in the enclosure depend on both the position of the cylinder and the Rayleigh number. Detailed discussion for the streamline, isotherms, and Nusselt number distribution is presented in this paper.

Lattice Boltzmann method, Cubical enclosure, Inner cylinder, Natural convection

I. INTRODUCTION

Over the past few decades, lattice Boltzmann method (LBM) has been used to describe the physical phenomenon of fluid. This method is based on the gas kinetic theory, can be characterized as a mesoscopic approach [1], and has successfully dealt with complex and dynamic boundary condition problems in fluid flow simulations including multiphase and multicomponent flows [1-6], micro-flows [6-9], turbulent flows [1, 6, 10, 11], and flows through porous media [2, 3, 12, 13].

Natural convection heat transfer from confined heat sources represents a fundamental problem in many engineering systems such as heat exchangers, heat pipes, electronic cooling devices, fluidized beds, and nuclear and chemical reactors. Moukalled and Acharya [14] numerically studied natural convection heat transfer from a heated horizontal cylinder placed concentrically inside a square enclosure. Shu and Zhu [15] studied the natural convection heat transfer efficiency in a concentric annulus between an outer square and a heated inner circular cylinder.

Most of these previous studies have investigated natural convection in an enclosure by using 2-D computational approach, which assumes that the enclosure has an infinite thickness. Nevertheless, natural convection phenomena in real space such as a cubical cavity cannot be correctly analyzed by 2-D simulation, as reported by Chan and Banerjee [16]. They studied the aspect ratio effect on natural convection in the hexagonal enclosure, and compared the results of 2-D and 3-D simulation where the 2-D simulation does not agree well with 3-D simulation, especially for the enclosure with an aspect ratio less than unity. Fusegi et al. [17] investigated 3-D steady natural convection in a cubical enclosure without an inner body. Ha and Jung [18] studied natural convection in a cubical enclosure with an inner heat-generating cubic conducting body. They found that as the Rayleigh number increases the flow and isothermal distributions change to complex three-dimensional patterns. Yoon et al. [19] investigated effects of the position of an inner sphere on natural convection in a cubical enclosure. They reported the local Nusselt number depends greatly on the position of the inner sphere as well as the Rayleigh number. Our group recently conducted a 3D finite-volume model to investigate plume behavior above a heated cylinder in water [20].

As shown in previous studies, natural convection in an enclosure depends on the geometry shape of enclosure and position of an inner body. Several experimental studies have been carried out in the last decade [21-25]. In the present study, the thermal lattice Boltzmann scheme proposed by Peng et al. [26] is used for simulating the cubical enclosure with different inner cylinder positions at the Rayleigh number of 10^4 to 10^6.

II. METHODOLOGY

A. The Thermal Lattice Boltzmann Method

The internal energy density distribution function approach proposed by He et al. [27] is used for simulating incompressible thermal flow. The governing equations are expressed as:

$$f_i(\vec{x} + \vec{e}_i\Delta t, t + \Delta t) - f_i(\vec{x}, t) = -\frac{1}{\tau_v}\left[f_i(\vec{x}, t) - f_i^{eq}(\vec{x}, t)\right] + \Delta t F_i$$

$$g_i(\vec{x} + \vec{e}_i\Delta t, t + \Delta t) - g_i(\vec{x}, t) = -\frac{1}{\tau_c}\left[g_i(\vec{x}, t) - g_i^{eq}(\vec{x}, t)\right] \quad (1)$$

where $f_i(\vec{x}, t)$ and $g_i(\vec{x}, t)$ are the particle density and internal energy distribution functions. $f_i^{eq}(\vec{x}, t)$ and $g_i^{eq}(\vec{x}, t)$ are the equilibrium particle density and equilibrium internal energy distribution functions associated with direction i at the position (x) and time (t). F is an external force term, $\vec{e}_i$ is the discrete velocity set, τ_v is relaxation times for the density

distribution function and τ_c is the relaxation time for internal energy distribution function.

Table I Comparison of the computed average Nusselt number on the middle plane of the cubical cavity and the central line in a 2-D cavity. Nature convection is induced by a single heated wall.

Ra	10^3	10^4	10^5	10^6
Davis (2D) [28]	1.118	2.243	4.519	8.800
Dixit and Babu (2D) [29]	1.121	2.256	4.546	8.652
Lin and Violi (2D) [30]	1.118	2.243	4.511	8.757
Perumal and Dass (3D) [31]	1.097	2.232	4.429	Non
Azwadi and Sidik (3D) [32]	1.096	2.301	4.67	Non
Peng et al.(3D) [26]	1.097	2.304	4.658	Non
Present TLBM (3D)	1.096	2.297	4.743	8.86

III. VALIDATION

Nusselt number is the most important dimensionless parameter that we can describe the convective heat transfer. The local heat flux along the x-direction $q_x(x,y)$ is equal to $u_x T - \alpha(\partial T/\partial x)$. The average Nusselt number at various planes can be calculated by employing a numerical integration procedure [33]:

$$\overline{Nu} = \frac{L}{\alpha \Delta T}\frac{1}{L^2}\int_0^L \int_0^L q_x(x,y)dxdy \quad (2)$$

Table I compares the simulation results from the present calculation with the literature results. The good agreement is seen between the TLBM and conventional CFD methods. Although not shown in the paper, maximum horizontal and vertical velocity on the mid-plane and the corresponding coordinates are also in excellent agreement with data reported in the literature.

IV. RESULTS AND DISCUSSION

Fig. 1 shows the cross-sectional view of the 3D model. All of the simulation conditions are the same as previous study [34, 35]. Seven location angles are chosen: 0°, 30°, 60°, 90°, 120°, 150° and 180°, in order to study the effect of cylinder locations on natural convection for different Rayleigh numbers in the enclosure.

Fig. 2 shows the distribution of the streamlines and isotherms when the inner cylinder moves rotationally around the enclosure center at Ra = 10^4. The dependence of the fluid flow and thermal fields in the enclosure on □□ can be observed in the change of streamlines and isotherms at different location angles with Ra = 10^4.

When the inner cylinder is located at the lowest position of □ = 0°, two overall rotating main eddies are respectively formed at the right and left sides of the mid x-plane of the enclosure. A weak upward thermal plume appears over the upper surface of the circular cylinder as shown in Fig 2(a). When the cylinder moves upward to · = 30°, the left eddy is bigger than right one and the thermal plume over the upper

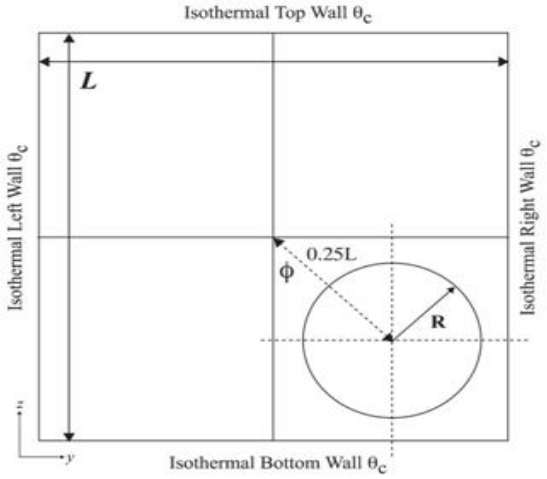
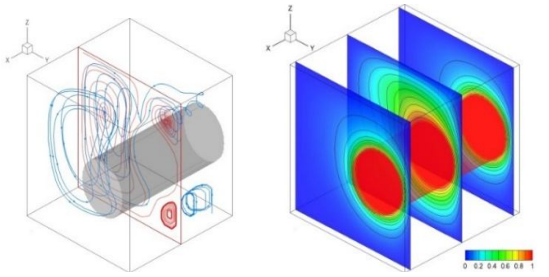


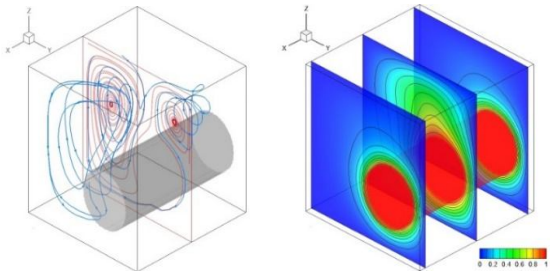
Fig. 1 Computational domain and boundary conditions for natural convection in the annulus between the eccentric circular cylinder and cubical walls.

surface of the circular cylinder is not as significant as the one at · = 0°. The plume grows over the upper surface of the cylinder but leans towards the left due to the wall confinement. At · = 60° (see Fig 2b), the right-hand side main eddy is divided into two small eddies, because the space between cylinder and the right wall is not enough for the fluid stream from the top to the bottom of the enclosure. The thermal plume at · = 60° clearly inclines to the left side. As the circular cylinder moves upward to · = 90° (see Fig 2c), the two small eddies have the similar size and the thermal plume is not more obvious than that at · < 90 °. The isotherms are only affected by the top wall of the enclosure.

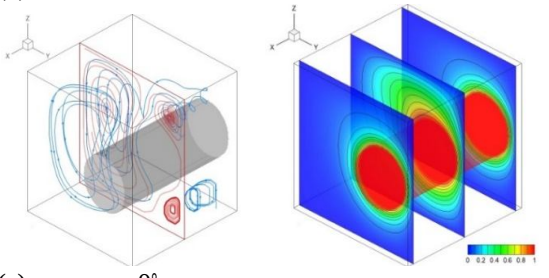
When the circular cylinder is placed at · = 120°, 150° and 180° (see Fig 2e-g), the two small eddies grow and merge into one when the space between the cylinder and right wall is increased. The upward thermal plume is not observed anymore owing to the decrease in the space between the circular cylinder and the wall of the enclosure. When the circular cylinder is close to the top wall of the enclosure, the clod and heavy fluid form a stagnation area in the lower half of the enclosure due to the strong convective flow that is confined in the cylinder surface and upper half of the enclosure. Fig. 3(a) shows the surface-averaged Nusselt number on the top wall of the enclosure. The value of Nu_T increases as the Rayleigh number increases regardless of the location of the inner circular cylinder. This characteristic is owing to the increased effect of buoyancy-induced convection. Nu_T values of the circular cylinder placed at the lower half of the enclosure, · ≤ 90°, have no significant difference between Ra = 10^4 and 10^5. However, when the cylinder is moved upward, the value of Nu_T increases with the height of the cylinder position.



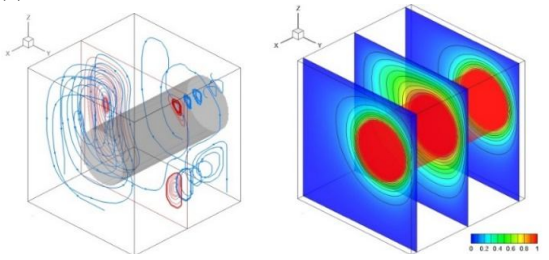
(a) □□□□0°



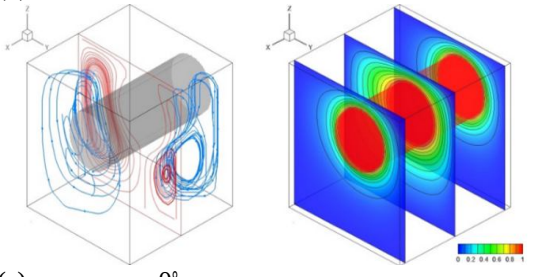
(b) • • • • • 0°



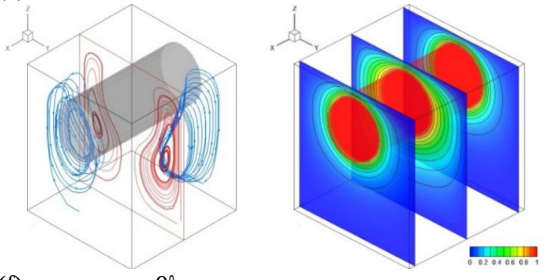
(c) • • • • • 0°



(d) • • • • • • 0°



(e) • • • • • • 0°



(f) • • • • • • 0°

Fig. 2 Isothermals and streamlines for different • • at Ra = 10^4.

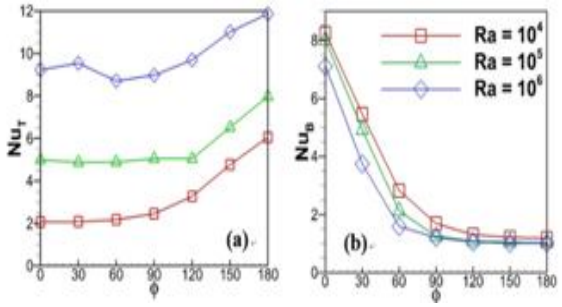


Fig. 3 Surface-averaged Nusselt number as a function of • for three different Rayleigh numbers on walls of the enclosure: (a) top wall and (b) bottom wall.

The difference of the Nu_T values between Ra = 10^4 and Ra = 10^5 decreases with an increase of • and becomes constant when the • is bigger than 120° (see Fig. 3(a)). As the distance between the upper surface of cylinder and the top wall of the enclosure decreases, the effect of convection on the fluid flow and thermal fields in the enclosure weakens. Thus, when • is larger than 120°, conduction is a leading mode of the heat transfer between the circular cylinder and the top wall of the enclosure.

When Ra = 10^6, the difference of Nu_T the values between • = 0° and 180° achieves a minimum. This is because when the Ra is high enough the effect of buoyancy-induced convection on the flow and thermal fields in the enclosure dominates; regard less of the position of the circular cylinder. When • is increased from 0° to 30°, as shown in Fig. 3(a), Nu_T increases gradually. However, when the circular cylinder is placed at • = 60°, Nu_T decreases because of the strong interaction between secondary vortices and the right wall, as shown in Fig. 3(a). Fig. 3(b) shows the surface-averaged Nusselt number on the bottom wall of the enclosure (Nu_B), as a function of • for different Rayleigh numbers. Regardless of the change of Rayleigh numbers, Nu_B decreases with an increase of •. When • ≤ 90°, the value of Nu_B decreases very rapidly with an increase of •. However, when • is varied between 90° and 180°, Nu_B decreases very slowly with an increased •. This is because the variation of isotherms in the lower half region of the enclosure is slow when the circular cylinder is placed in the range of upper half enclosure.

CONCLUSIONS

In this research, we investigate the natural convection in a cubical enclosure with inner heated circular cylinder located on different positions along the gravitational direction and analyze the difference between 2-D and 3-D simulations. The position of the inner cylinder not only influences the growth of thermal plume but also affects surface-average Nusselt numbers.

The effect of the 3-D confinement on natural convection is presented with three different planes. The middle plane of the 3-D model is similar to the

2-D simulation results in terms of structure of streamlines and isotherms. Along the cylinder axial line, the Nusselt number on the cylinder surface near the enclosure wall is lower compared with that near the middle of the enclosure. In particular, the heat transfer between the cylinder and fluids near the enclosure wall is dominated by heat conduction. So, the 3-D simulation is necessary to study natural convection above the heated cylinder placed in an enclosure.

The Rayleigh number is the most important parameter in this simulation. It uses to control the conduction and convection behavior of heat transfer in the enclosure. When the Rayleigh number increases, the convection becomes dominant.

For the future, our in-house code could be extended to validate experimental data.

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