NONLINEAR OPTIMAL PID CONTROLLER DESIGN USING DIRECT TRANSCRIPTION METHOD FOR A HALF-CAR ACTIVE VEHICLE SUSPENSION SYSTEM (AVSS) IN PRESENCE OF ROAD DISTURBANCES

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Abstract—Satisfying costumers' requirements is the key factor in maintaining market share of large automaker companies. In this manner, one of the most important features is isolating passengers from annoyance of vibrations initiated from road surface roughness. At the same moment, ride quality and steerability are other major aspects that must not be sacrificed. As a result, a great deal of effort is made to improve suspension system of cars to achieve this goal. In this paper, authors have modeled a half-car active vehicle suspension system(AVSS) with new features and acceptable accuracy. We have proposed an optimal control method based on direct transcription to control a half-car AVSS in presence of road disturbances. Simulation results show desirable performance in controlling important parameters.

Index Terms- active vehicle suspension system (AVSS), feedback, half-car model, nonlinear control, Direct Transcription, nonlinear, optimal, PID controller.

I. INTRODUCTION

Satisfying costumers' requirements is the key factor in maintaining market share of large automaker companies from which ride comfort is one of the most important. As a result, a great deal of effort is made to improve suspension system of cars to achieve this goal.

Car suspension system, logically, is responsible for two main functions [1]:

- Absorbing oscillations and vibrations imparted to wheels from road surface roughness
- Providing effective contact between wheels and road surface

In designing suspension system of cars, driving quality and steer ability are two main factors that are hard to be satisfied simultaneously; e.g. by increasing each one, the second is deteriorated.

From irrational point of view, suspension system is designed to dampen vibrations transmitted to body of car and passengers. Therefore, three types of systems are distinguishable: passive, semi-active and active suspension systems [2].

As expected, AVSSs are the most sophisticated. In this type, damping coefficient of damper and spring hardness are adjustable. An electrically controlled hydraulic actuator is added to the combination as the control leverage.

In AVSSs, information about vertical accelerations of car wheel and body, and vertical position of each wheel relative to body is measured by accelerometer and position sensors and transmitted to Electronic Control Unit (ECU). ECU controls the system via forces exerted by electro hydraulic actuator which results in better damping of vibrations from road surface roughness. In other words, external forces are checked and control signals are made according to information from sensor to keep the car desirably steerable and stable. As results, dampers of car can perform with different damping coefficients [3].

Several attempts were done to design a suitable AVSS in recent years. All these attempts are based on a mathematical model and implementing various controller on. Modeling is the first step in development, optimization, and improvement of performance of a dynamic system. Without sufficient knowledge, manipulation of system is likely to result in unacceptably high risks. Forecasting dynamic behavior of a system, on the other hand, provides invaluable opportunities for improving its performance and avoiding common mistakes. These two, collectively, play the main role in success of a dynamic system design.

Control methods have witnessed a gigantic growth thanks to development of cars, robots and other high-tech systems. Controller is the heart of all control system which is responsible for signaling suitable inputs to gain desired outputs. A good controller features acceptable stability, perturbation isolation, low sensitivity to noise and variations in parameters, and following reference input. It tries to maintain control parameters in a predefined range. Car suspension system, as a specialty, is nonlinear and time-variant, and its parameters tend to vary over time.

There are several methods for controlling car suspension systems which can be divided into two main categories: linear and nonlinear. Regarding nonlinear nature of car over rough road surfaces, it is obvious that modern control methods offer more

accurate results over classic methods [4].

Direct transcription method is one of the best optimal options due to its flexibility, and easy implementation of dynamic constraints on state and control variables. In this method, a suitable cost function is defined to determine control variables. Therefore, it is possible to attain ride comfort, steering quality and controllability at the best possible level in presence of dynamic and differential constraints.

Optimal control method are generally divided into two main categories [5]:

- Indirect methods
- Direct methods

In direct methods, state and control variables are computed directly (with no need to extra variables and parameters). Meanwhile, indirect methods are based on using peripheral variables and parameters in the first step (like pseudo-state variables) and then state and control variables are computed from them. Both methods strive to change optimal control method into a standard optimization problem (e.g. nonlinear programming) [6]. Standard optimization problems lack dynamicism (contrary to optimal control) and their optimization variables are time-independent. This property helps to use versatile and potent tools in solving the problem.

II. LITERATURE REVIEW

Up to now, several control methods are applied to car AVSS. Chien et. al. [7] and Huang et. al. [8] used feedback linearization for a multiple input- multiple output (MIMO) in designing car AVSS controller (respectively, for whole and half car models). Nonlinearity of the system, generally, owes to rolling and pitching in large angles about their centers of gravity. In these two works, dynamics of actuator are neglected and all components of suspension system are regarded to be linear. Huang et. al. improved FBL method by combining it with Fuzzy Logic Control (FCL).

Fateh and Alavi designed impedance controller to compromise between ride comfort and road holding. They implemented their method on a quarter-car AVSS with a nonlinear hydraulic actuator. A couple of control loops is used: FBL for controlling hydraulic actuator force in internal loop and FCL for controlling mass-spring position in external loop. Ruth-Horwitz criterion is the main tool used by them [9].

Renn and Wu [10] utilized two control schemes for nonlinear model of quarter-car with servo-hydraulic actuator. They firstly devised a discrete-time PID controller and adjusted control gains using Ziegler-Nichols rules. The second schemes was based on a neural network controller trained by gradient steepest descent back propagation method.

Deshpande and colleagues proposed a new scheme to reduce acceleration of sprung mass using sliding mode control. In their method, effects of uncertain nonlinear spring and damper, load variations, and unknown road disturbances is estimated. This scheme only depends on states of sprung mass as input and is augmented by a disturbance observer based sliding mode control scheme for active suspension [11].

Wang et.al. chose car's body vertical vibration acceleration on a quarter-car model as their control goal. They initially, used a fuzzy PID controller based on cultural algorithm to improve their scheme, but in was shown that a combination of cultural and niche algorithms results in better performance. They concluded that an AVSS controlled by a fuzzy PID controller (with rules improves by cultural algorithm) entails a significantly better performance in suppressing vertical acceleration of body and higher ride comfort [12].

Chen et.al. presented an improved optimal sliding mode (SM) for a car nonlinear AVSS to obtain both true nominal optimal suspension performance and better robustness. They used linearized feedforward and feedback method and shown that optimal sliding mode controller entails no better performance compared to existing methods. As a remedy chien et.al. used an advanced edition of sliding mode composed of an augmented optimal sliding mode manifold function. This new scheme had more robust characteristics [13].

We, in this paper, have chosen ride comfort and steerability as the main aspects of stress regarding environmental constraints. Following sections are respectively allocated to modeling assumptions, nonlinear modeling of suspension, tire force modeling, input disturbance modeling, hydraulic actuator modeling, and nonlinear AVSS modeling with hydraulic actuator dynamics. Then formulation of problem, the solution and architecture of the proposed controller using direct transcription method is brought with results of simulations demonstrated.

III. MODELING ASSUMPTIONS

We have assumed following properties in mathematical modeling of a half-car nonlinear AVSS [14]:

1) All connections and fittings in suspension system are regarded ideal.
2) The car is cruising rectilinearly with a constant speed. We have tested various speeds to measure robustness of controller toward variations in speed. All forces and torques are neglected to simplify behavior of braking.
3) Both sprung and un-sprung masses are assumed to be equal.
4) Loading of sprung mass do not vary over time. We have applied various loads over sprung mass to measure controller's robustness toward variations in this parameters.

5) Car body is modeled rigid and non-elastic.
6) Roughness and irregularities of road surface are the sole sources of car vibration (elastic deformation of road and vibrations of engine are neglected).
7) Non-uniformity of wheels and instability are not included.
8) Spring center of mass is situated on central line of car body.
9) Sensors are used to react to variations in measured parameters.
10) Heaving and pitching are only main movements of car body. Other probable movements (e.g. yawing and rocking) and lateral movements of gravitational mass are neglected.
11) Simulations are done around dynamic equilibrium point. Therefore, mass of car is not considered.
12) Angular displacement of car body varies and in large angles is assumed about equilibrium point.
13) Dampers connect wheels to car body moving at wheel-hop frequency. Therefore their effect is neglected.

IV. NONLINEAR MODELING OF SUSPENSION SYSTEM

An active car suspension system is illustrated in figure (1) incorporating main components. Using free diagram in figure (1) suspension force is estimable [14].

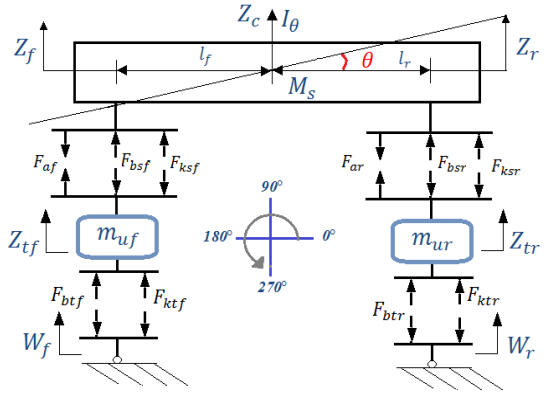


Fig 1. Free body diagram of a half-car AVSS

While angular displacement of pitch θ can vary to large angles, it is assumed around equilibrium point. Therefore, displacements of front and rear sprung masses z_f and z_r are as below. Velocities of front and rear sprung masses are respectively $\dot{z}_f$ and $\dot{z}_r$ which are time derivatives of displacements.

$$z_f = z_c - l_f Sin\theta \quad (1)$$

$$\dot{z}_f = \dot{z}_c - l_f \dot{\theta}\ Cos\theta \quad (2)$$

$$z_r = z_c + l_r Sin\theta \quad (3)$$

$$\dot{z}_r = \dot{z}_c + l_r \dot{\theta}\ Cos\theta \quad (4)$$

In which l_f and l_r are distance between front and rear wheels from gravitational center of sprung mass and $\dot{\theta}$ is angular velocity of pitch.

Suspension springs are linear: ks_f^l and ks_r^l mention front and rear springs while nonlinear constants of ks_f^{nl} and ks_r^{nl} usher to front and rear nonlinear spring constants. Regarding relative displacement between front and rear un-sprung masses, spring force of suspension system is calculated using k_{ksf} and F_{ksr}.

$$F_{ksf} = ks_f^l (z_{tf} - z_f) + ks_f^{nl} (z_{tf} - z_f)^3 \quad (5)$$

$$F_{ksr} = ks_r^l (z_{tr} - z_r) + ks_r^{nl} (z_{tr} - z_r)^3 \quad (6)$$

In which z_{tf} and z_{tr} are displacements of front and rear sprung masses, respectively. Suspension dampers involve asymmetric components of bs_p^{nl} and bs_r^{nl}. Regarding relative velocity between un-sprung and sprung masses, F_{bsf} and F_{bsr} (front and rear damping forces) are introduced in (7) and (8).

$$F_{bsf} = bs_f^l (\dot{z}_{tf} - \dot{z}_f) - bs_f^{sym} |\dot{z}_{tf} - \dot{z}_f| + bs_f^{nl} \left(\sqrt{|\dot{z}_{tf} - \dot{z}_f|}\right) \mathrm{sgn}(\dot{z}_{tf} - \dot{z}_f) \quad (7)$$

$$F_{bsr} = bs_r^l (\dot{z}_{tr} - \dot{z}_r) - bs_r^{sym} |\dot{z}_{tr} - \dot{z}_r| + bs_r^{nl} \left(\sqrt{|\dot{z}_{tr} - \dot{z}_r|}\right) \mathrm{sgn}(\dot{z}_{tr} - \dot{z}_r) \quad (8)$$

For which $\dot{z}_{tf}$ and $\dot{z}_{tr}$ are respectively velocities of front and rear un-sprung masses and "sgn" refers to signum function.

V. TIRE FORCE MODELING

Tire damping coefficient is a function of its type, friction, force exerted on, and it state (free or spinning). Due to complexity of estimating tire damping, it is generally ignored in most researches. In this research, damping of front and rear wheels vibrations are demonstrated with bt_f and bt_r:

$$bt_f = 0.1(bs_f^l) \quad (9)$$

$$bt_r = 0.1(bs_r^l) \quad (10)$$

Tire force in front and rear section (F_{tf} and F_{tr}) equals sum of forces related to tire spring and damping. Front tire force components of F_{ktf} and

F_{btf} in (11) and (12) are spring and damping forces while the same is true for F_{ktr} and F_{btr} about rear wheel, respectively (in (14) and (15)). K_{tf} and K_{tr} are spring coefficients of front and rear springs.

$$F_{ktf} = kt_f(z_{tf} - \omega_f) \quad 11$$

$$F_{btf} = bt_f(\dot{z}_{tf} - \dot{\omega}_f) \quad 12$$

$$F_{tf} = F_{ktf} + F_{btf} \quad 13$$

$$F_{ktr} = kr_r(z_{tr} - \omega_r) \quad 14$$

$$F_{btr} = bt_r(\dot{z}_{tr} - \dot{\omega}_r) \quad 15$$

$$F_{tr} = F_{ktr} + F_{btr} \quad 16$$

Input disturbance models for front and rear wheels ω_r and ω_f are presented in (11) and (14) and developed in following section while $\dot{\omega}_f$ and $\dot{\omega}_r$ are respectively their time derivatives.

VI. INPUT DISTURBANCE MODELING

Car vibrations are mainly due following sources:

1) Gusts and aerodynamic properties
2) Un damped/ weakly damped engine
3) Road surface properties
4) Car state in maneuver (pitch, brake, acceleration)

In this research, road surface properties is regarded as the sole source of vibration.

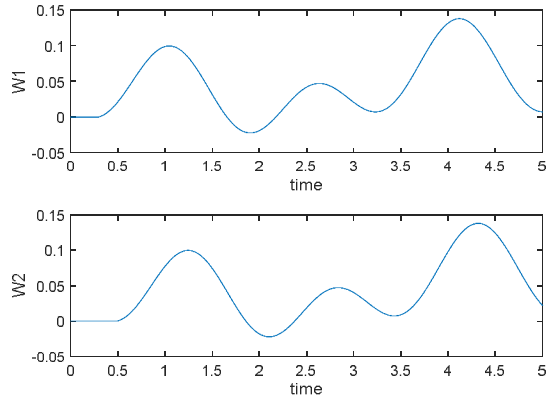


Fig.2. Road input disturbance.

VII. HYDRAULIC ACTUATOR MODELING

In mentioned dynamic equations, the force in suspension system is provided by a hydraulic actuator. The magnitude of hydraulic force F_{ai} (i stands for front (f) or rear (r)) is determined from spools and three sections in a four-way manifold. These spools are designed to work in range of $|x_i| \le u_{\max}$ in which is $u_{\max}$ maximum acceptable input voltage. For an input magnitude x_i, is the rate of spool displacement $\dot{x}_{vi}$ which is stated with following equations:

$$\dot{x}_{vi} = \frac{1}{T}(-x_{vi} + Kv_i u_i) \quad 17$$

Which is a linear filter with time constant τ for hydraulic actuator. In the notation we have x_{vi} as spool displacement, and K_{vi} for transformation ration of input voltage to spool displacement per meter.

Hydraulic load current Q_i and pressure drop in piston PL_i is defined as below:

$$Q_i = \text{sgn}[P_s - \text{sgn}(x_{vi})Pl_i]C_{dis}\Omega x_{vi} \sqrt{\frac{1}{\rho}|P_s - \text{sgn}(x_{vi})Pl_i|} \quad 18$$

$$\dot{Pl}_i = \frac{4\beta_e}{V_t}\left[Q_i - C_{tp}Pl_i - A_{hyd}(\dot{z}_i - \dot{z}_{ti})\right] \quad 19$$

In which hydraulic pressure source is demonstrated with P_s, discharge coefficient with C_{dis}, gradient of spool surface with Ω, effective volume module with β_e, total actuator volume with V_t, and bleeding coefficient with C_{tp}, hydraulic piston area with A_{hyd}, and hydraulic fluid density with P. equations (24) and (25) are simplified using α, β and γ as below:

$$\alpha = \frac{4\beta_e}{V_t} \quad 20$$

$$\beta = \alpha C_{tp} \quad 21$$

$$\gamma = \alpha C_{dis}\Omega x_{vi}\sqrt{\frac{1}{\rho}} \quad 22$$

It is possible to rewrite these equations as follows:

$$Q_i = \text{sgn}[P_s - \text{sgn}(x_{vi})Pl_i]\sqrt{|P_s - \text{sgn}(x_{vi})Pl_i|} \quad 23$$

$$\dot{Pl}_i = \gamma Q_i x_{vi} - \beta Pl_i - \alpha A_{hyd}(\dot{z}_i - \dot{z}_{ti}) \quad 24$$

Therefore, hydraulic force F_{ai} with hydraulic piston area A_{hyd} are related with (31):

$$F_{ai} = A_{hyd}Pl_i \quad 25$$

Table 1. Parameters of hydraulic actuator

Parameters	Magnitude
α	$4.515\times10^3\ N/m^{-5}$
β	$1s^{-1}$
γ	$1.545\times10^9\ N/m^{5/2}/kg^{1/2}$
piston area (A_{hyd})	$3.35\times10^{-4}\ m^2$

barometer (P_s)	10342500 Pa
time constant (τ)	$1/30s$
spool gains (K_{vf} and K_{vr})	$0.001\,m/v$

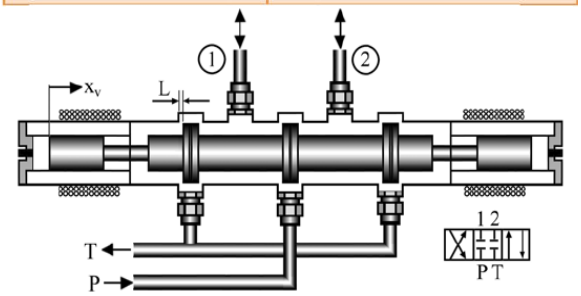


Fig 3. Hydraulic actuator

VIII. NONLINEAR AVSS WITH DYNAMICS OF HYDRAULIC ACTUATOR

Combining forces of suspension springs and dampers and hydraulic actuator forces, it is possible to calculate forces for front and rear wheels. We have:

$$f_f = F_{ksf} + F_{bsf} - F_{af} \quad (26)$$

$$f_r = F_{ksr} + F_{bsr} - F_{ar} \quad (27)$$

Therefore, equations of motion for half-car nonlinear AVSS with dynamics of hydraulic actuator are derivable using Newton's second law of motion. From figure 1 for a car massed M_s we have:

$$\ddot{z}_c = \frac{1}{Ms}\left[f_f + f_r\right] \quad (28)$$

$$\ddot{\theta} = \frac{1}{I_\theta}\left[-l_f\, f_f Cos\theta + l_r\, f_r Cos\theta\right] \quad (29)$$

$$\ddot{z}_{tf} = \frac{1}{mu_f}\left[-F_{tr} - f_f\right] \quad (30)$$

$$\ddot{z}_{tr} = \frac{1}{mu_r}\left[-F_{tr} - f_r\right] \quad (31)$$

For which $\ddot{z}_c$, $\ddot{z}_{tf}$ and $\ddot{z}_{tr}$ are respectively accelerations of gravitational center of sprung mass, front un-sprung mass, and rear un-sprung mass. $\ddot{\theta}$ and I_θ show angular acceleration of body and moment of inertial in pitch. mu_f and mu_r are front and rear un-sprung masses.

IX. FORMULATING CONTROLLER DESIGN PROBLEM USING DIRECT TRANSCRIPTION OPTIMAL CONTROL METHOD

In each optimal control problem, the goal is to attain to an optimal state. This state is definable in a scalar cost function (performance index function) which usually takes a form like below [15]:

$$J = h(x,t) + \int g(x,u,t)dt \quad (32)$$

Equation (32) is said to have Bolza form and includes Mayer and Lagrange terms. Mayer term entails a function of problem variables and their derivatives while Lagrange term is composed of problem variables and their time derivatives in a time interval. For any optimal control problem it is assumed that system constraints are first-order ordinary differential equations, initial and final boundary conditions, and other constraints related to state variables and controllers. A short description of each case is brought in follow. Differential equations have a general form of:

$$\dot{\bar{x}} = f(\bar{x},\bar{u},t) \quad (33)$$

Other optimal control constraints are defined based on physical limitations, intentions of designers, and limitations of vehicle.

It is also possible to determine, confine and partition permissible ranges of variation for problem variables in a predefined interval or in other words, an upper and lower bound for each variable. Variables may be bounded (both sides, or just one) or unbounded. Physical problems are constrained by physical limitations of problem. We have:

$$X_l(\tau) \le X(\tau) \le X_u(\tau) \quad (34)$$

We aim to solve the formulated problem in the following section using direct transcription optimal control method.

X. SOLVING PROBLEM USING DIRECT TRANSCRIPTION OPTIMAL CONTROL METHOD

Optimal control dates back to Brachistocorone [1] problem proposed by Jacob Bernoulli in 17th century and calculus of variations. Theory and framework of optimal control is usually recalled as dynamic optimizations and as a result, it is possible to optimize dynamic systems under an index of performance, using it.

In a number of references, solving methods of optimal control problems are divided into two main groups: direct and indirect. A detailed description of these methods is accessible in [15].

In this method, a continuous optimal control problem is discretized to form discrete algebraic equations instead of differential and integral equations. Here, an optimal control problem is converted to a standard ordinary optimization problem in nonlinear programming and solved with much easier methods. Additionally, there is no further need to integral them. The initial time interval is divided into several smaller intervals in the first step. Each time interval is an essentially a node. Intervals can be produced even or

[1] The shortest time in greek

uneven, but for more ease, there are usually chosen in equal distances. For n nodes and time intervals h_k we have:

$$h_k = \frac{t_f - t_i}{n-1} \quad (35)$$

In solving optimal control problems using direct transcription method, it is sufficient to find magnitudes of state and controls in each node. For this, each one is regarded as a variable. To estimate equations in each node, there are several methods like trapezoidal, Hermit, Simpson, et.al.

By using each method, optimal control problem becomes a static optimization problem. Finally, a myriad of methods are available to solve the nonlinear programming problem.

XI. ARCHITECTURE OF PROPOSED CONTROLLER DESIGNED WITH DIRECT TRANSCRIPTION

Controller architecture:

Controller architecture for AVSS composed of two control loops is as below:

- Internal loop, force control
- External loop, suspension control

This controller architecture is illustrated in figure 1. External control loop is tasked to find suitable force to feed hydraulic actuator. Meanwhile, internal loop assures magnitude of applied force.

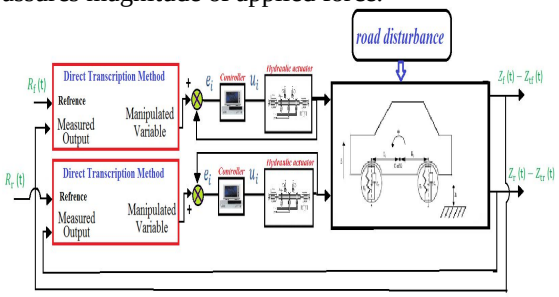


Fig 4. Control architecture for a nonlinear active vehicle suspension via direct transcription method and PID force feedback.

- PID control loop design

Control voltage of hydraulic actuator u_i (from external loop) is brought in equation 18. External (force) loop in demonstrated in figure 2. The applied controller architecture is similar to [26-28].

$$u_i = K_p \times (e) + K_d \times de(t)/dt + K_p \int (e)dt \quad (36)$$

$$e_i = F_{airef} - F_{ai} \quad (37)$$

In which K_{pi}, K_{Ii} and K_{di} are coefficients of proportional, integral and differential gains coefficients. e_i is defined in equation 21 as error between reference and real actuator forces. Letter i is used for both front and rear suspensions. Saturation block in front of controller output is devised to observe $(|u_i| \leq |u_{max}|)$ constraint. Control gains of internal loops are tabulated in table 1.

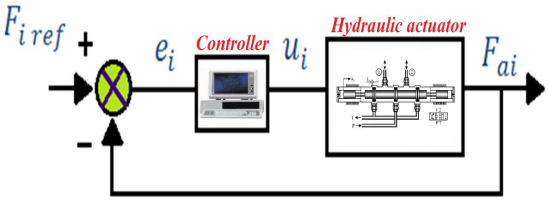


Fig 5. Internal nonlinear optimal PID force feedback control loop.

- PID suspension control loop design

Reference desirable actuator force signal F_{airef} is produced by suspension control loop with structure similar to [14].

$$F_{airef} = K_{pi} \times e_i(t) + K_{Ii} \int e_i(t)dt + K_{Di} \frac{de_i(t)}{d_t} \quad (38)$$

$$e_i = Z_{ti} - Z_i \quad (39)$$

For which Ri and ei are reference signal and control error, respectively. Here again, letter i is used for both front and rear suspensions. It is assumed that $R_i = 0$). Therefore, for $e_i \rightarrow 0$ we have $t \rightarrow \infty$ [14]. Control gains are presented in table 1. There are three main goals in design:

- Attaining a better performance for AVSS for decreasing RMS of control parameters.
- Designing a controller robust to variations in load of sprung mass and cruise velocity of car.
- Designing a controller robust to the roughness of road surface and adaptable to various environment conditions [14].

To reach these goals, the authors have used direct transcription method to find control constants. Figures … and … illustrate results for designed nonlinear optimal PID controller using this method.

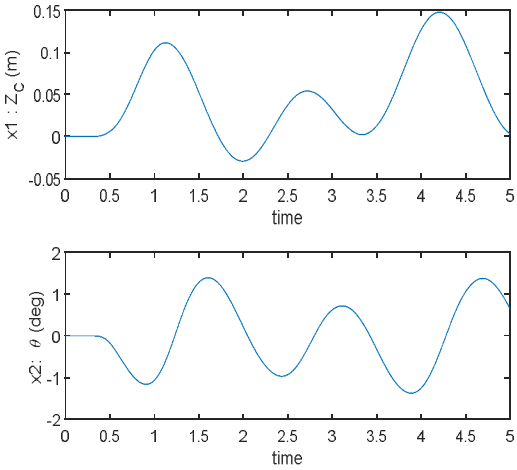


Fig. 6. Displacements of equilibrium point of sprung mass and pitch angular.

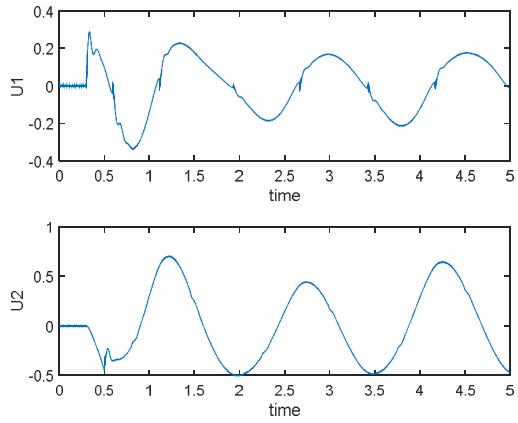


Fig. 7: Front and rear actuator control voltage.

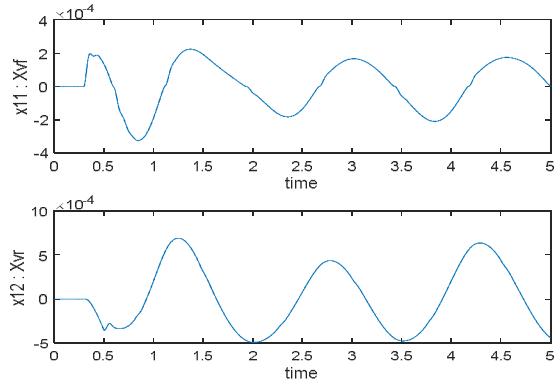


Fig. 8. Servo valves displacements.

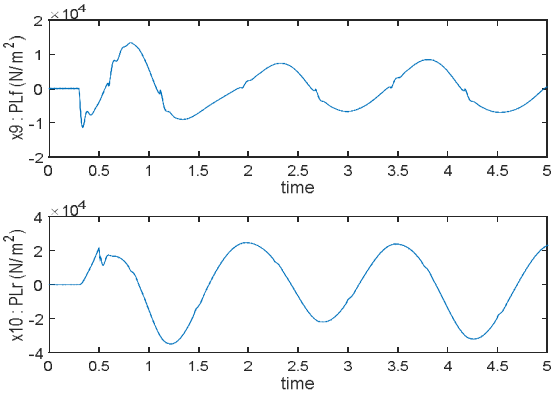


Fig. 9. Front and rear actuator force.

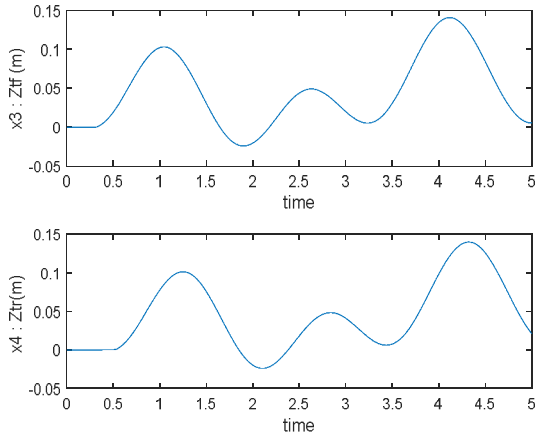


Fig. 10. Front and rear sprung mass displacements.

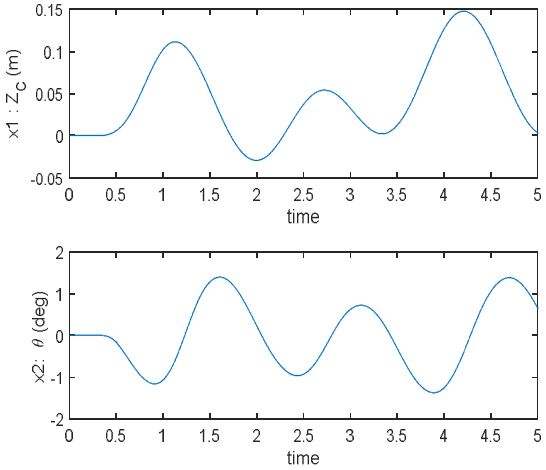


Fig. 11. Displacements of equilibrium point of sprung mass and pitch angular.

CONCLUSION

The authors aim to find parameters for optimal control in an AVSS with nonlinear components. The controller satisfies physical constraints of hydraulic actuator in permitted range of variance for dynamic variables, controllers and temporal conditions.

In the first step, modeling of suspension system is presented. Then nonlinear optimal control problem of suspension system in presence of road disturbances is formulated using direct … method. In the third step, the process to convert optimal control problem into a nonlinear programming problem is shown and the proposed architecture of controller presented. Finally, results of simulating the system in MATLAB® is presented.

As seen, the proposed control system initially tries to pass the minimum extent of disturbances to the passengers. By facing other disturbances over road surface, the controller manages to adjust parameters accordingly to provide the highest level of ride comfort.

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