ON THE DYNAMIC ANALYSIS OF FREE-FLYING AND FREE-FLOATING BEHAVIORS OF A FLEXIBLE SPACE ROBOT

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Abstract—The dynamic behavior of a space robot, considering free-flying and free-floating modes of operation for the shuttle/spacecraft motion, is studied in this paper. For the free-flying space manipulator, the governing equations of motion are derived by means of Lagrange's approach. For the free-floating mode, Kane's method is used to facilitate deriving the equations governing the nonholonomic behavior of the robot manipulator. Assumed mode method, incorporating quasi-comparison functions, is employed to express elastic displacements of the flexible arms. For both space robots, the resulting nonlinear problem is separated into two sets of equations by a perturbation approach; one for rigid-body maneuvering of the robot and the other for elastic vibrations and rigid-body perturbations. The kinematic redundancy of the free-flying space robot is removed, assuming that attitude of the spacecraft is controlled during the maneuvering and conservation of linear momentum is governed on the rigid robot. Accordingly, path planning of the free-flying space robot can be accomplished analytically. In the free-floating mode, the kinematic redundancy is removed by imposing both conservation of linear and angular momentums on the manipulator system. Since the conservation of angular momentum is nonintegrable, the free-floating robot manipulator behaves as a nonholonomicsystem; hereby the trajectory tracking in the free-floating mode is done numerically. A numerical example is presented to establish a comparison between the dynamic behaviors of the two robotic systems.

Index Terms—Space robot, Kane's dynamics,Lagrange's dynamics,free-flying, free-floating.

I. INTRODUCTION

In recent few decades, a broad investigation has been carried out in the area of space technologies.Robotic systems have been proposed for crucial missions in various fields, such as nuclear and toxic waste cleanup, construction and infrastructure, medical hospitals, and of course, for space missions.Among these significant developments, space robotics has received a great deal of attention as the new technological field of robotics by many researchers.The necessity of deploying robotic systems to perform various tasks in space has been realized over these years.For instance, these robots can be used for the collection of space debris, refueling large spacecrafts, autonomous target chase and capture, construction and maintenance of space stations, repairing and servicing satellites in earth orbits, and exploring various objectives in space.

According to the types of the platform motion, the space robot has been classified into free-flying and free-floating robots.In a free-flying manipulator system, the position and attitude of the system base, i.e. the spacecraft, isactively controlled by reaction jets to perform a prescribed maneuvering task.For instance, jet thrusters can be chosen as force actuators and reaction wheels (RWs) may be used as torque actuators.In contrast, in free-floating space robotic system, the platform or spacecraft's position and attitude are not actively controlled to conserve attitude control fuel.The free-floating mode of operation is of interest for space robots for several reasons among which saving attitude control fuel and facilitating repair missions are key advantages.

To maximize the workspace of space manipulator systems, the manipulator arms should be rationally long.On the other hand, the arms must be relatively light due to weight limitations.Considering both features, the manipulator arms must be highly flexible.Despite of ground-fixed manipulators with flexible arms, for which there are numerous modeling and controlworks, researches on free-flying or free-floating robots with flexible arms are not too many.Meirovitch, Stemple, and Kwak [1] took a three-dimensional space robot with serially connected flexible arms and derived its equations of motion by using a Lagrangian approach in term of quasi-coordinates.The robot base was able to translate relative to a platform, which itself can translate and rotate in the inertial space.By assuming that the elastic motion and rigid-body perturbations are small in comparison with rigid-body maneuvering motions, the equations were linearized.Based on the linearized equations, a perturbation scheme was used to design the control laws for vibration suppression of the robot.Meirovitch and Lim [2] utilized Lagrange's dynamics to derive the equations of motion of a free-flying space robot including hinge-connected flexible arms.Based on the assumption that elastic vibrations is one order of magnitude smaller than the rigid-body maneuvering, a perturbation approach permits design of controls for the two types of motion separately.To suppress elastic vibrations and rigid-body perturbations, a discrete-time linear quadratic regulator with prescribed degree of stability was applied.Chen and Meirovitch [3] analyzed a flexible space robot executing a docking maneuver with a target whose motion is not known a priori.Lagrange's equations were used to derive its equations of motion and then separate them into two

sets of equations suitable for rigid-body maneuver and vibration suppression control.For the rigid-body maneuver, on-line feedback tracking control was carried out by means of an algorithm based on Lyapunov-like methodology and using on-line measurements of the target motion.For the vibration suppression, LQR feedback control in conjunction with disturbance compensation was carried out by the use of collocated sensor/actuator pairs dispersed along the flexible arms.Stemple [4] presented an efficient technique for generating equations of motion for flexible multibody structures, carried out by means of the Principle of Virtual Work, often referred to as d'Alembert's Principle in the dynamics literature.Multibody dynamics equations were used to simulate the controlled motion of a flexible space robot, such as the one used on the space shuttle. The results were presented in the form of plots of the various joint displacements, the displacements of points on the beams due to the elastic motion, as well as the control torques of the various actuators.Ukai, Asano, and Morita [5] presented a distributed parameter modelling and the non-holonomic kinematics of a free-floating space robot with flexible arms maneuvering in a horizontal plane.Both the Lagrange's method and the Hamilton's principle were used to take account of the translational momentum conservation law. The relation between non-holonomic constraint and elastic oscillations were discussed by using infinite dimensional systems.They also considered the tracking control problem and investigated the asymptotic stability of the sensor's feedback control scheme.Romano [6] has studied the problem of tracking a reference trajectory, in the joint space, and suppressing structural vibrations for manipulators that have flexible links, as it typically happens in space robotics applications.A manipulator system, consisting of several links connected by means of actuated revolute joints was considered in the research work.Highly nonlinear and coupled dynamic equations were derived, and in the contemplated open kinematic chain, some of the links were considered as Euler-Bernoulli beams to model their flexibility.

The robot considered in this paper consists of a rigid platform that can be a shuttle or a spacecraft, carrying the other parts of the robot; two hinged-connected flexible arms, and an end-effector (such as a gripper), performing a desired task such as moving a payload.The rigid platform can move in x- and y-directions while rotating about the z-axis.The flexible arms can undergo rotation about their hinged position with previous body along with elastic deformation, and the end-effector can be subjected to rotation.Figure 1 shows the kinematic arrangement of the space robot linkages, including all the dimension details.

For the free-flying mode of operation, the equations of motion are derived by means of Lagrange's approach, presented by Meirovitch and Lim [2].The derivation is based on consistent kinematical synthesis, i.e. motion of each body in the chain takes into consideration the motion of the preceding body in the chain, according which the imposition of constraints in the equations of motion is eliminated.The obtained equations are nonlinear and of relatively high order.In order to solve the equations of motion, and by relying on the fact that in this space robot, elastic motions are small compared to the rigid-body maneuvering motions, it is convenient to use a perturbation approach to linearize them.Such an approach leads to separation of the problem into a zero-order problem for the rigid-body maneuvering of the space robot and a first-order problem for the control of the elastic motions and the perturbations from the rigid-body motions [7].The rigid-body maneuvering of the space robot can be done through an open-loop scheme using inverse kinematics and dynamics but the elastic motions and rigid-body perturbations require closed-loop control.Path planning of the space robot consists of picking an object, carrying out this payload over a desired trajectory and placing it in a certain position with specific orientation.

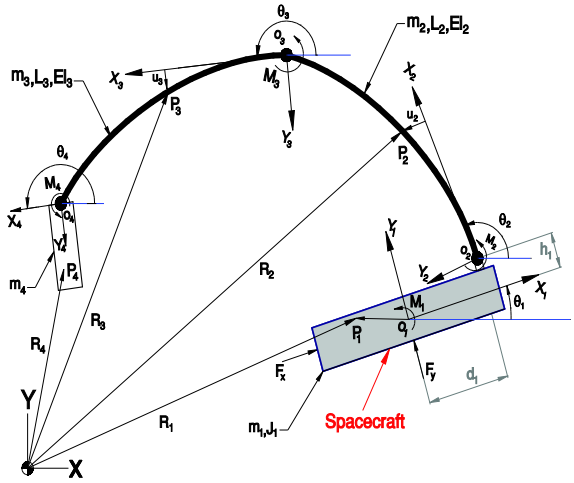


Fig.1. A schematic of the four-link space robot, considering free-flying and free-floating modes of the spacecraft motion.

In the free-floating manipulator system, due to the nonintegrability of the conservation of angular momentum equation, nonholonomic behavior is observed, which means that the spacecraft orientation is not a function of the current joint configuration only, but also a function of the desired path.In other words, in these manipulator systems the spacecraft is permitted to translate and rotate in response to manipulator motions.Furthermore, the system degrees of freedom is degenerate, due to the kinematic singularities, and is effectively less than original degrees of freedom of the system.It is important to note that the singularities in free-floating robots are no longer a function of the robot kinematic and become dependent on the dynamic properties of the robot as well.Free-floating mode of operation for the same robot has been studied by Masoudi [8].In the presence

of non-holonomic constraints, it is more efficient to derive equations of motion by means of Kane's method.The resultant equations are simply a set of ordinary differential equations (ODEs), rather than a set of differential and algebraic equations (DAEs), and therefore can be solved more conveniently.The obtained equations are solved by perturbation approach, as in the case of free-flying robot, and divided into zero-order and first-order equations.Due to existence of linear and angular momentum conservation laws, and for a prescribed initial condition of the robot configuration, the free-floating robot is not kinematically redundant.However, since the conservation of angular momentum equation is not integrable, the trajectory planning equations must be solved numerically. To this end, Multi-step Gear method has been applied [9].Estimating the time history of joints motion, which is computed by means of the numerical method, the forces and torques are obtained using the zero-order equations.The flexible arms have been modeled by assumed mode method, incorporating quasi-comparison functions instead of admissible functions used in the approximation scheme [7].Assuming that the flexible arms act as Euler-Bernoulli beams, the use of such functions are admissible.These comparison functions satisfy geometric compatibility along with bending moments and shearing forces at boundary points.Considering the flexible arms as cantilever beams with nonzero shearing force and bending moment at the boundaries, the quasi-comparison functions can be obtained in the form of suitable linear combinations of clamped-free and clamped-clamped shape functions.The dynamic behavior of the space robot for the two modes of operation, i.e. free-flying and free-floating forms, in a particular mission can be compared.

Energy and fuel economy due to free-floating mode of operation, effect of applying a control theory on elastic vibrations, and considering more complicated missions are some potential and challenging topics for future works.

GOVERNING EQUATIONS OF MOTION

A. Discrete Modeling of the Flexible Arms

Assuming that the flexible arms act as Euler-Bernoulli beams, the elastic displacements can be written as [2]

$$u_{i(x_i,t)} = \sum_{j=1}^{n_i} \phi_{ij}(x_i)\eta_{ij}(t) = \boldsymbol{\phi}_i^T(x_i)\boldsymbol{\eta}_i(t), i = 2,3 \quad (1)$$

where $\phi_{ij}(x_i)$ and $\eta_{ij}(t)$, $i = 2,3$; $j = 1,2,\dots,n_i$ are *shape functions* and *generalized coordinates*, respectively, and n_i is the number of summation utilized in modeling the i^{th} link; $\boldsymbol{\phi}_i$ is n_i-dimensional space-dependent vector containing admissible functions and $\boldsymbol{\eta}_i$ is n_i-dimensional time-dependent vector containing generalized coordinates.It should be noted that geometric boundary conditions along with natural boundary conditions must be satisfied when defining the shape functions in the discrete modeling of the flexible arms.The flexible arms are tangent to axis x_i at o_i, hence the shape functions and their slope must be zero at $x_i = 0$.At $x_i = L_i$, the displacement, slope, bending moment, and shear force are generally nonzero.To satisfy the above-mentioned conditions, a particular class of functions called *quasi-comparison functions* is utilized [2].The quasi-comparison functions are defined as linear combinations of admissible functions capable of approximating the differential equation and the natural boundary conditions to any degree of accuracy by merely increasing the number of terms applied in the approximating solution [2].The admissible functions must be selected so that a linear combination of them possesses the above boundary characteristics of the flexible links in the manipulator system.Thereupon, suitable quasi-comparison functions consist of linear combinations of clamped-free and clamped-clamped shape functions, or

$$\phi_{ik}(x_i) = \frac{1}{\sqrt{L_i}}\left[\cosh\frac{\lambda_k x_i}{L_i} - \cos\frac{\lambda_k x_i}{L_i} - \sigma_k\left(\sinh\frac{\lambda_k x_i}{L_i} - \sinh\frac{\lambda_k x_i}{L_i}\right)\right] \quad (2)$$

which represent the eigenfunctions of clamped-free beams or clamped-clamped beams. $L_i$is the length of each arm, x_i is the longitudinal component of the arm-fixed axis, and λ_k and σ_k are nondimensional parameters.

B. Free-Flying Space Manipulator System

Lagrangian mechanics is the scheme applied for thederivation of equations of motion in case of free-flying modeof operation for the space robot. The same solution procedure,presented by former researchers [7], [2], is applied inthis work to regenerate the dynamic model for simulatingthe free-flying manipulator system.

Consistent kinematicalsynthesis along with a perturbation approach, assuming thatelastic vibrations are small compared to the rigid-bodymaneuvering motions, are used to extract equations of motionand derive zero-order and first-order equations. These equations,respectively, are related to the rigid-body maneuvering,and elastic vibrations together with perturbations from therigid-body motion. The generalized coordinate vector for themanipulator system shown in Fig. 1 can be defined as

$$\boldsymbol{q}(t) = \left(X(t)\ Y(t)\theta_1(t)\theta_2(t)\theta_3(t)\theta_4(t)\eta_2^T(t)\ \eta_3^T(t)\right)^T \quad (3)$$

in which $(X(t)\ Y(t))$ is the position vector of the origin o_1 of the shuttle frame axes, which is fixed to the center of mass of the shuttle, in terms of inertial coordinate, $\theta_i(t)$ is the rotation angle of the i^{th} reference frame attached to the i^{th} manipulator link, measured relative to the inertial frame, and $n_2 = n_3 = n$ for the flexible links. Accordingly, the Lagrange's equations of motion can be expressed as

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\boldsymbol{q}}}\right) - \frac{\partial L}{\partial \boldsymbol{q}} = \boldsymbol{Q} \quad (4)$$

where$L$ is the Lagrangian of the manipulator system, representing the difference between kinetic energy and potential energy of the whole system, i.e.

$$L(\boldsymbol{q}, \dot{\boldsymbol{q}}) = T(\boldsymbol{q}, \dot{\boldsymbol{q}}) - V(\boldsymbol{q}) \quad (5)$$

Kinetic energy of the system is defined as

$$T = \frac{1}{2}\sum_{i=1}^{4}\int_{m_i} \boldsymbol{V}_i^T \boldsymbol{V}_i d_{m_i} = \frac{1}{2}\dot{\boldsymbol{q}}^T M \dot{\boldsymbol{q}} \quad (6)$$

in which the velocity vector $\boldsymbol{V_i}$ represents the velocity of an infinitesimal mass on the i^{th} linkage and$M$ is the mass matrix, extracted from the integration of the infinitesimal kinetic energies of the whole system.Potential energy, which is due to the deformation of the flexible linkages, can be expressed as

$$V = \frac{1}{2}\sum_{i=2}^{3}\int_{L_i} EI_i\left(\frac{\partial^2 u_i}{\partial x_i^2}\right)^2 dx_i = \frac{1}{2}\boldsymbol{q}^T K \boldsymbol{q} \quad (7)$$

with$K$ as the stiffness matrix for the whole system, derived by introducting the discrete flexible beams model, i.e. (2), into the potential function.$\vec{Q}$in(4) is the generalized force vector, acquired from virtual work due to all the external loads (forces/torques) applied to the manipulator system.

$$\delta W = \vec{Q}^T \delta \boldsymbol{q} \quad (8)$$

The detailed information for all the above matrices and vectors can be found in [2] and [7].

Applying Lagrange's dynamic equations, i.e.(4), the differential equations governing the dynamic behavior of the space robot can be represented as

$$M\,\ddot{\boldsymbol{q}} + \left(\dot{M} - \frac{1}{2}\dot{\boldsymbol{q}}^T \frac{\partial M}{\partial q}\right)\dot{\boldsymbol{q}} + K\boldsymbol{q} = \vec{Q} \quad (9)$$

C. Free-Floating Space Manipulator System

In this section, free-floating mode of operation for the same flexible manipulator system is considered.

Accordingly, a comparison analysis between its dynamic behavior and the one for the free-flying space robot would be presented.As mentioned before, applying Kane's method in the derivation of the equations of motion is quite efficient for this mode of operation due to nonholonomic constraint present in the spacecraft motion.The mission and parameters of the free-floating space robot are the same as those in the free-flying space robot except that we have no control on both position and orientation of the spacecraft.The consistent kinematical synthesis is utilized in derivation of the equations of motion and a perturbation approach is used to linearize and separate them into zero-order and first-order equations.Spatial discretization of flexible links is accomplished by means of the Assumed Mode Method presented in SectionII-A.

In Kane's scheme, if we define generalized coordinates as

$$\widehat{\boldsymbol{q}}(t) = \left(\theta_1(t)\theta_2(t)\theta_3(t)\theta_4(t)\eta_2^T(t)\eta_3^T(t)\right)^T \quad (10)$$

ther$^{\text{th}}$generalized speed u_r is defined in the following form:

$$u_r \triangleq \dot{\hat{q}}_r, \quad r = 1, 2, \ldots, 4 + [2n] \quad (11)$$

and, accordingly, partial angular velocities ω_j^r and partial velocities V_j^{*r} for the j^{th} linkage can be estimates as

$$V_j^* = \sum_{r=1}^{4+2n} V_j^{*r} u_r \text{ and} \omega_j = \sum_{r=1}^{4+2n} \omega_j^r u_r \quad j = 1,4 \quad (12)$$

in which V_j^* and ω_j are respectively the velocity of center of mass and angular velocity of the j^{th}link with respect to the inertial frame.

So, the general form of the Kane's equation is written as

$$F_r + F_r^{\text{Flexible arms}} + F_r^*\Big|_{\text{Rigid links}} + F_r^*\Big|_{\text{Flexible arms}} = 0,$$

$r = 1, 2, 3, \ldots, 4 + 2n$ (13)

where F_r is the r^{th} generalized active force, representing the contribution of all actuator torques to the input power to the manipulator system, corresponding to the r^{th} generalized speed [8], [10], $F_r^{\text{Flexible arms}}$, the contribution to generalized active forces due to flexibility, can be obtained from potential energy function V as

$$F_r^{\text{Flexible arms}} = F_r \Big|_{\text{Due to link flexibility}} = -\frac{\partial V}{\partial \hat{q}_r}, \quad r = 1, 2, \ldots, 4 + 2n \tag{14}$$

The generalized inertia forces, F_r^*, associated with the generalized speed u_r are divided into rigid links generalized inertia forces and flexible arms generalized inertia forces. The former is expressed as

$$F_r^*\Big|_{\text{Rigid links}} = V_1^{*r}.(-m_1 A_1^*) + \omega_1^{*r}(-J_1\alpha_1) + V_4^{*r}.(-m_4 A_4^*) + \omega_4^{*r}(-J_4\alpha_4) \tag{15}$$

where α_1 and α_4 are absolute angular accelerations of 1^{st} and 4^{th} links, respectively, and $A_j^*,\ j = 1, 4$ is the acceleration vector of center of mass of the j^{th} link,and the latter can be written as

$$F_r^*\Big|_{\text{Flexible arms}} = \int_{m_2} V_2^r.(-A_2)\, dm_2 + \int_{m_3} V_3^r.(-A_3)\, dm_3 \tag{16}$$

in which $A_2,\ j = 2, 3$ is the acceleration vector of a point on the j^{th} flexible link.Equation (13) can be written in matrix form as follows

$$M(\hat{\boldsymbol{q}})\ddot{\hat{\boldsymbol{q}}} + h\left(\hat{\boldsymbol{q}}, \dot{\hat{\boldsymbol{q}}}\right) = \vec{Q} \tag{17}$$

where $M(\hat{\boldsymbol{q}})$ is $(4 + 2n) \times (4 + 2n) -$ dimensional mass matrix and $h\left(\hat{\boldsymbol{q}}, \dot{\hat{\boldsymbol{q}}}\right)$ is $(4 + 2n) -$dimensional vector.

D. Perturbation Approach on the Dynamical Equations

The obtained system of motion equations is high-order, nonlinear, and coupled. Hence, a perturbation approach is applied to separate the obtained equations into zero-order and first-order equations, considering that the elastic vibrations are one order of magnitude smaller than the maneuvering motions. The zero-order equations are used for rigid-body maneuvering of the space robot and the first-order system can be used for the control of the elastic motions and perturbations from the rigid-body motion.

Applying a perturbation scheme on the state variables and generalized force vectors of the equations of motion, we assume

$$\boldsymbol{q}(t) = \boldsymbol{q}_0(t) + \epsilon\, \boldsymbol{q}_1(t) \text{ and } \vec{Q}(t) = \vec{Q}_0(t) + \epsilon\, \vec{Q}_1(t) \tag{18}$$

where the subscripts 0 and 1 denote the zero-order and first-order quantities, respectively. Using the Taylor series expansion, the zero-order equations can be extracted as

$$M_0\ddot{\boldsymbol{q}}_0 + h(\boldsymbol{q}_0, \dot{\boldsymbol{q}}_0) = \vec{Q}_0 \tag{19}$$

in which $M_0 = M(\boldsymbol{q}_0)$. The first-order equations are

$$M_0\ddot{\boldsymbol{q}}_1 + M_v\dot{\boldsymbol{q}}_1 + M_q\boldsymbol{q}_1 = \vec{Q}_1 + \vec{Q}_d \tag{20}$$

in which coefficient matrices M_v and M_q are derived from Taylor series expansion of the equations of motion and applying perturbation relations(18)$\vec{Q}_d$ is the persistent disturbance vector, obtained by substituting zero-order variables into the left side of the equation of motion.It is obvious that the obtained coefficients are different for the considered free-flying and free-floating space robots [7], [8], [10].

TRAJECTORY PLANNING OF THE SPACE ROBOT

In trajectory planning, the robot should be considered as a manipulator system whose parts are all rigid.The key point in trajectory planning is calculating the joints position and velocity profiles along a prescribed path between the given initial and final configurations of the end-effector.In other words, describing the manipulator motion regardless of the actions that cause the motion is considered as trajectory planning.

E. Free-flying Space Robot

The equations relating end-effector configuration, i.e, end-effector position and angle, in Cartesian space and joint positions of the space manipulator can be written in differential form using direct differential kinematics as below

$$\dot{X}_{E_0} = J(\boldsymbol{q}_0)\dot{\boldsymbol{q}}_0 \tag{21}$$

where X_{E_0} is the zero-order position and angle vector of the manipulator end-effector during the space robot

maneuvering in Cartesian space, and $J(\boldsymbol{q}_0)$ is the 3×6Jacobian matrix mapping linearly the zero-order joint velocity into Cartesian velocity.An analytical solution has been derived by Lim and Meirovitch [1] [7], by partitioning the generalized coordinates as

$$\boldsymbol{q}_0 = \left(\boldsymbol{q}_s^T \vdots \boldsymbol{q}_m^T\right)^T \quad (22)$$

where $\boldsymbol{q}_s = (X_0 Y_0 \theta_{10})^T$ and $\boldsymbol{q}_m = (\theta_{20}\theta_{30}\theta_{40})^T$. They also assumed that no forces act on the space robot platform during the zero-order (rigid-body) maneuvering. So, linear momentum of the robot system is conserved as there are no thrusters in the trajectory planning, or

$$\dot{\boldsymbol{q}}_s = -\frac{1}{m_t} C_m \dot{\boldsymbol{q}}_m \quad (23)$$

Substituting (23) in (21), the trajectory planning equations can be generated in terms of $\dot{\boldsymbol{q}}_m$ only as

$$\dot{X}_{E_0} = -\frac{1}{m_t} C_m \dot{\boldsymbol{q}}_m + J_m \dot{\boldsymbol{q}}_m \rightarrow \dot{X}_{E_0} = J_C \dot{\boldsymbol{q}}_m \quad (24)$$

where $J_{C\,(3\times3)}$ is the condensed Jacobian matrix.Accordingly, an analytical solutionto estimate all the joint variablescan be derivedto perform the inverse kinematics.

F. Free-floating Space Robot

In case of free-floating space robot, no reaction wheels or controlling torques act on the system in the trajectory-planning phase.Therefore, both conservation of linear and angular momentum are governing constraint equations on the space system.Similar to the one that we had in free-flying space robot, the payload possesses three degrees of freedom and the space robot has six degrees of freedom.It means that the number of joint space variables is more than the number of task space variables. It may seem that kinematic redundancy exists, but since conservation of linear and angular momentum impose three constraint equations on motion of the rigid free-floating robot, the number of equations and unknowns will be the same and kinematic redundancy will not exist.However, conservation of angular momentum is not integrable and we are not able to solve the equations associated with trajectory tracking, analytically.To cope with the problem, a numerical scheme is applied. Equations derived using *direct differential kinematics* and *conservation of linear momentum*, along with the *conservation of angular momentum* as

$$J_1\dot{\theta}_{10} + m_1\left(-\dot{X}Y + \dot{Y}X\right) + J_2\dot{\theta}_{20} + m_2\left(-V_{\mathrm{cmx}_2}Y_{\mathrm{cm}_2} + V_{\mathrm{cmy}_2}X_{\mathrm{cm}_2}\right) + J_3\dot{\theta}_{30} + m_3\left(-V_{\mathrm{cmx}_3}Y_{\mathrm{cm}_3} + V_{\mathrm{cm3}}X_{\mathrm{cm}_3}\right) + J_4\dot{\theta}_{40} + m_4\left(-V_{\mathrm{cmx}_4}Y_{\mathrm{cm}_4} + V_{\mathrm{cm}_4}X_{\mathrm{cm}_4}\right) = 0 \quad (25)$$

are assembled to represent joint space variables for prescribed values of task space variables in the following differential form [8]:

$$A(\boldsymbol{q}_0)\frac{d\boldsymbol{q}_0}{dt} = B\left(X_{E_0}\right)$$
$$\boldsymbol{q}_0(0) = \left(X_0(0)Y_0(0)\theta_{10}(0)\theta_{20}(0)\theta_{30}(0)\theta_{40}(0)\right)^T \quad (26)$$

Where$A_{6\times6}$ is a time-dependent matrix, and$B_{6\times1}$ is a time-dependent forcing vector.In (25), J_i is the polar moment of inertia of i^{th}link about its canter of mass, $V_{\mathrm{cmx_i}}$ and $V_{\mathrm{cmy_i}}$ are the absolute velocity vector components of center of mass of i^{th} link in X and Y directions, respectively, and $X_{\mathrm{cm_i}}$ and $Y_{\mathrm{cm_i}}$ are the position vector components of center of mass of i^{th} link.The above system of differential equations is solved numerically in order to determine the amounts of joint space variables for prescribed values of task space variables and known initial conditions of joint space variables, i.e. $\boldsymbol{q}_0(0)$.

SIMULATION RESULTS AND DISCUSSION

In this work, the mission considered for both free-flying and free-floating space robot is to carry out a desired payload in the Cartesian space in a straight-line maneuver.The initial and final positions of the end-effector are selected to avoid kinematic redundancy, which occur in either fully stretched or fully folded arm configurations.The initial and final end-effector positions are defined as [2].

$$X_{E_{i0}} = \left(9.757\ \text{m}\ \ 1.914\ \text{m}\ \ \frac{3\pi}{2}\ \text{rad}\right)^T$$
$$X_{E_{f0}} = (5.000\ \text{m}\ \ 1.914\ \text{m}\ \ \pi\ \text{rad})^T \quad (27)$$

The center of mass of the entire manipulator system is at $(0.995\ \text{m}\ 0.191\ \text{m})$ in the initial configuration.It is important to note that the center of mass position is invariant during the rigid-body maneuvering.An inventory of numerical values for the parameters used in the simulation of the flexible space robots (see Fig. 1) is provided in Table 1.The maneuvering time is chosen $t_f = 2.5$ s but to show the vibration behavior of the system due to the links'flexibility, it is extended to $t = 3.5$ s.

Link 1	Link 2	Link 3	Link 4
m_1 = 10.0 Kg	m_2 = 1.0 Kg	m_3 = 1.0 Kg	m_4 = 0.1 Kg
h_1 = 10.0 m	L_2 = 5.0 m	L_3 = 5.0 m	L_4 = 1.66 m
J_1 = 20.0 Kg. m^2	EI_2 = 122.28.0 N. m^2	EI_3 = 122.28.0 N. m^2	J_4 = 3.0 Kg. m^2

Table 1. The parameters for linkages of the free-flying and free-floating space robots.

Applying inverse kinematics and conservation of linear momentum (along with conservation of angular momentum for the free-floating mode of operation), joint positions, velocities, and accelerations are computed for the prescribed maneuver.Accordingly, the zero-order actual actuator forces and torques for the trajectory tracking of the rigid space robots are evaluated using zero-order equations of motion (19).As shown in Fig. 2, the thruster forces on the spacecraft are identically zero in case of free-flying mode of operation.For the free-floating robot, the thruster forces and torque on the spacecraft are zero, as expected.

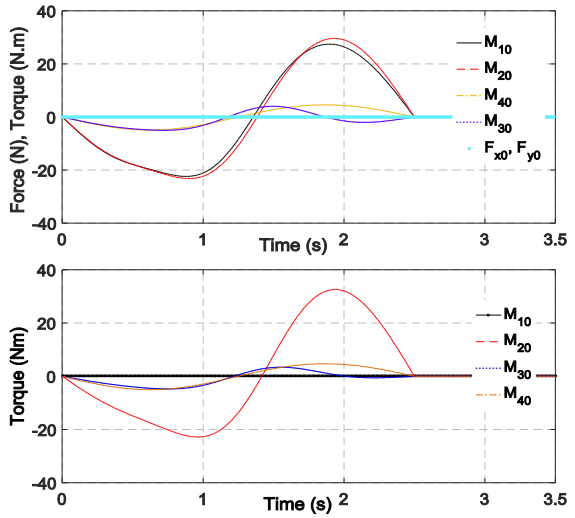


Fig. 2. Time history of zero-order actuator forces/torques for free-flying (top) and free-floating (bottom) space robots.

Rotation of the robot spacecraft for free-flying and free-floating modes of operation during maneuvering of the manipulator system is demonstrated in Fig. 3.

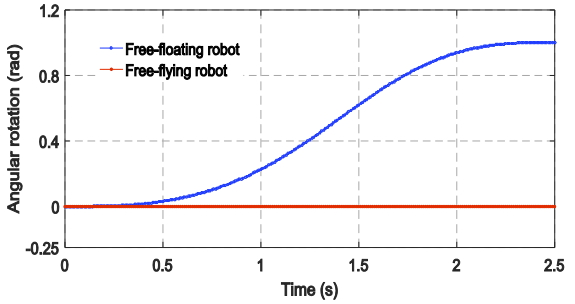


Fig. 3. Time history of the spacecraft rotation for free-flying and free-floating space robots.

As it is obvious, for free-flying space robot, the spacecraft orientation is zero due to zero-order torque M_{10} applied on the spacecraft (see Fig.2).In contrast, for the free-floating mode of operation, the spacecraft angular position varies due to its dynamic interaction with the rest parts of the manipulator system, which is generated from conservation of angular momentum relation.It means that the spacecraft orientation is not a function of the current joint configuration only, but also a function of the desired path.

Figures4 and 5 show position errors in x- and y-direction and orientation errors of the end effector and spacecraft about z-axis for both modes of operation of the space robot.The errors are due to perturbations in rigid-body motions and vibrations of flexible links.As expected, in the free-floating mode of operation for the space robot, the attitude control fuel may be saved which in turn increases the robot life in space.However, the position and orientation errors due to the links flexibility and perturbations in rigid-body motions are higher in the free-floating robot, which requires more energy to suppress the vibrations and perturbations.

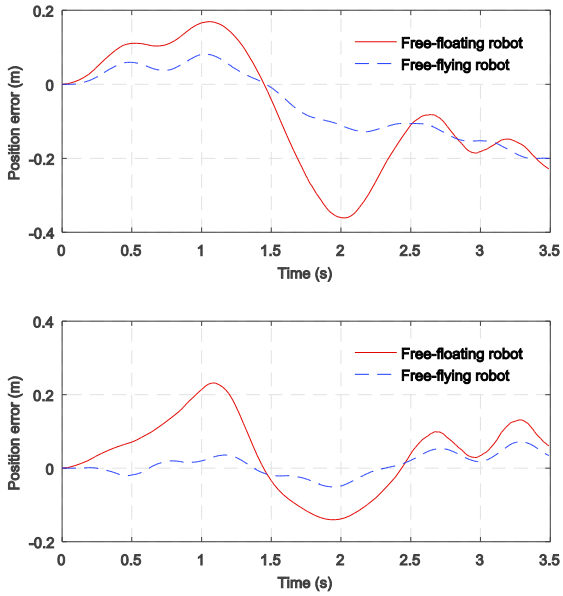


Fig. 4.Time history of the end-effector position error in the x-direction (top) and y-direction (bottom) for free-flying and free-floating modes of operation.

Exploring the efficacy of applying a control scheme to both space robots and comparing the results provide a challenging context for future works.

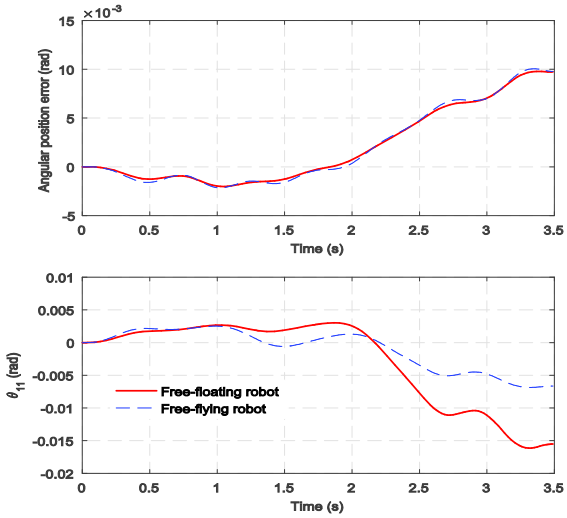


Fig. 5.Time history of end-effector rotation error about z-axis (top) and perturbations in rigid-body rotation of the spacecraft (bottom) for free-flying and free-floating modes of operation.

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